Forecast Trading as a Means to Reach Social Optimum on a Peer-to-Peer Market[1]

Ilia Shilov[1,3,*], Hélène Le Cadre[2], Ana Bušić[1], Anibal Sanjab[3], and Pierre Pinson[4]

[1] Inria Paris, DI ENS, CNRS, PSL University, France
ilia.shilov@inria.fr
[2] Université de Lille, Inria, CNRS, Centrale Lille, UMR 9189 - CRIStAL, France
[3] VITO/EnergyVille, Thorpark 8310, 3600 Genk, Belgium
[4] Imperial College London, Dyson School of Design Engineering, London, UK

Abstract. This paper investigates the coupling between a peer-to-peer (P2P) electricity market and a forecast market to alleviate the uncertainty faced by prosumers regarding their renewable energy sources (RES) generation. The work generalizes the analysis from Gaussian-distributed RES production to arbitrary distributions. The P2P trading is modeled as a generalized Nash equilibrium problem, where prosumers trade energy in a decentralized manner. Each agent has the option to purchase a forecast on the forecast market before trading on the electricity market. We establish conditions on arbitrary probability density functions (pdfs) under which the prosumers have incentives to purchase forecasts on the forecast market. Connected with the previous results, this allows us to prove the economic efficiency of the P2P electricity market, i.e., that a social optimum can be reached among the prosumers.

Keywords: Mechanism Design · Decentralized Electricity Market · Peer-to-peer Market · Forecast Market

1 Introduction

Decentralization in electricity markets, driven by liberalization, the increase in renewable energy sources (RES), and the growing role of prosumers, has given rise to P2P energy markets, in which prosumers negotiate with each other for energy procurement while minimizing their costs and accounting for the uncertainties in RES generation. This decentralization can lead to more efficient and flexible energy distribution Le Cadre et al. [2020]. However, managing the uncertainties associated with RES generation remains a significant challenge Perakis and Roels [2008], Nair et al. [2014]. In traditional energy markets, uncertainties faced by end-users are handled by larger entities such as suppliers/retailers, who can mitigate these uncertainties by managing a large portfolio of users Moret

[1] This paper extends our previous work Shilov et al. [2023] on the integration of forecast markets with peer-to-peer (P2P) electricity trading by generalizing from Gaussian to arbitrary distributions.

et al. [2020]. However, in decentralized electricity markets, agents must accommodate the uncertainty in their generation profiles caused by RES, relying on their forecasts of future uncertain outcomes. Therefore, forecasting is crucial for making informed decisions in such markets. This aspect has been widely studied in the literature (see e.g. Petropoulos et al. [2022] for a detailed overview). The possibility to improve the forecasts needed, using external available data or forecast mechanisms, leads to the concept of information (or data) markets Agarwal et al. [2019]. Forecast markets, which aggregate and distribute information about uncertain future events, have shown promise in improving forecast quality Wolfers and Zitzewitz [2004], Messner and Pinson [2019]. These markets reward forecasters based on the accuracy of their predictions and their contribution to improving the client's utility Raja et al. [2023], Lambert et al. [2008].

Previous research Shilov et al. [2023] introduced a novel coupling of forecast markets with P2P electricity markets, using Gaussian distributions to model RES generation uncertainties. This coupling allowed prosumers to purchase forecasts in the forecast market modeled after Lambert et al. [2008], Raja et al. [2023] to improve their trading decisions in the P2P market. The P2P trading was modeled as a generalized Nash equilibrium problem (GNEP) Harker [1991], where prosumers' second-stage decisions depend on their forecasts of RES generation. This research demonstrated that the impact of forecast updates on a prosumer's outcome can be evaluated independently, allowing one to internalize the utility caused by the forecast update. Furthermore, Shilov et al. [2023] illustrated that the economic efficiency of the electricity P2P market can be achieved if prosumers participate in the forecast market. It was shown that the coupling is individually rational for Gaussian distribution-based forecasts, alongside intuition and numerical evaluation for the general case.

This paper extends the findings of Shilov et al. [2023] to accommodate arbitrary distributions. We generalize the conditions under which market efficiency and individual rationality are achieved when prosumers participate in the forecast market. Extending the theoretical framework to arbitrary distributions, we establish conditions under which agents have incentives to participate in the forecast market, ensuring individual rationality. It is remarkable that, for one-shot game, these conditions depend on the local properties of the forecasts, i.e. on the local shape of the distributions. These findings allow us to expand the previous results supporting the development of efficient P2P markets.

2 Model

2.1 Peer-to-Peer Electricity Market

We rely on a two-settlement electricity market design consisting of day-ahead and balancing (real-time) markets. We assume the presence of a backup retailer from whom the community can purchase energy both in day-ahead (hereafter, referred to as first stage) and in real-time (hereafter, referred to as second stage). Therefore, we fix the buying (b) and selling (s) prices for first (or day-ahead p_{da}) and second (or real time p_{rt}) stages, such that $p^{rt,b} > p^{da,b} > p^{da,s} > p^{rt,s}$. The

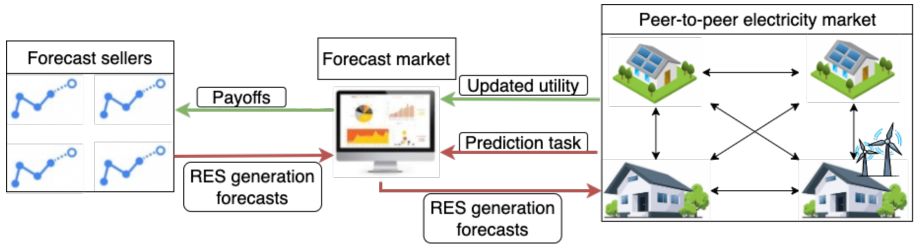


Fig. 1: Proposed framework overview: red arrows indicate pre-electricity market clearing actions, green indicate post-clearing.

community is seen as a price-taker in the electricity market, hence making the prices exogenous to this problem, similarly to the model considered in Moret et al. [2020].

Let $\Gamma_i \subseteq \mathcal{N}$ denote the set of neighbors of agent i, which reflects the agents with whom she wants to trade. We denote the trade between agent i and $j \in \Gamma_i$ as q_{ij} (limited with upper-limit κ_{ij}), where q_{ij} is the amount of power i purchases from (or sells to) j if $q_{ij} \geq 0$ ($q_{ij} \leq 0$) and impose a bilateral trading reciprocity constraint $q_{ij} + q_{ji} = 0$. Trading cost term is presented in the cost function as $\sum_{j \in \Gamma_i} c_{ij} q_{ij}$, where parameters $c_{ij} > 0$ represent (product) differentiation prices and reflect agent i's preferences for energy trading. Denote d_i as agent i's demand and Δg_i as agent i's renewable energy generation (wind, solar, etc.) which we assume to be a random variable with a CDF, $F_r \in [0, \infty)$. In this work, we do not account for a correlation between agents' random variables while it constitutes an important step for a further research. Then, each agent has to make a trading decision in the first stage (day-ahead market) about acquiring ($q_i^{da,b}$) or selling ($q_i^{da,s}$) energy at prices $p^{da,b}$, $p^{da,s}$ respectively. At the second stage (real-time market), agents settle imbalances after observing the realization of Δg_i for the prices $p^{rt,b}$ (buying) and $p^{rt,s}$ (selling).

$$
\begin{aligned}
\min_{q_i^{da}, q_i^{rt}, \mathbf{q}_i} \quad & \overbrace{p^{da,b} q_i^{da,b} - p^{da,s} q_i^{da,s} + \sum_{j \in \Gamma_i} c_{ij} q_{ij}}^{1^{st} \text{ stage costs}} + \mathbb{E}\Big[\overbrace{p^{rt,b} q_i^{rt,b} - p^{rt,s} q_i^{rt,s}}^{2^{nd} \text{ stage costs}} \Big] \\
\text{s.t.} \quad & q_{ij} + q_{ji} = 0, \qquad \forall j \in \Gamma_i && \text{(1a)} \\
& d_i = \Delta g_i + \sum_{j \in \Gamma_i} q_{ij} + q_i^{da,b} - q_i^{da,s} + q_i^{rt,b} - q_i^{rt,s} && \text{(1b)} \\
& q_{ij} \leq \kappa_{ij}, \qquad \forall j \in \Gamma_i && \text{(1c)} \\
& q_i^{da,b} \geq 0, q_i^{da,s} \geq 0, q_i^{rt,b} \geq 0, q_i^{rt,s} \geq 0 && \text{(1d)}
\end{aligned}
$$

Note, that the expectation of the second stage costs is taken with respect to a distribution with CDF, F_r, which represents a real distribution of Δg_i. Nevertheless, without full knowledge of this distribution, agent i has access to a forecast (belief) with CDF F_i about the distribution of Δg_i, which she uses for computing the solution of the problem (1). Detailed description and motivation for the model can be found in Shilov et al. [2023].

2.2 Coupled market model

For the forecast market we adopt a model from Raja et al. [2023], Lambert et al. [2008]. As authors show in [4], this forecast market mechanism enjoys some desirable properties such as budget-balance, anonymity, sybilproofness, truthfulness for the client and an individually rational (IR) for the forecast sellers, adapted from Lambert et al. [2008]. Note that in our study we investigate IR of the buyers of the forecasts, which is different from the latter. We refer to Raja et al. [2023], Lambert et al. [2008] and Shilov et al. [2023] for more details.

Efficiency of the Peer-to-Peer Market Optimal procurement quantities for the agents we derived in Shilov et al. [2023] by solving problem (1) as a variant of stochastic inventory management problem complicated by the peer-to-peer trading. Below we provide some results from Shilov et al. [2023] concerning the solution and efficiency of the peer-to-peer market that we will use later for the discussion on individual rationality. First, denote residual after first-stage decisions as $r_i := d_i - q_i^{da,b} + q_i^{da,s} - \sum_{j\in\Gamma_i} q_{ij}$ and note that it is non-negative. We derive the closed-form expression of the optimal procurement strategy in the presence of day-ahead and real-time contracts in a market with random renewable generation is

Theorem 1 (Shilov et al. [2023]). *The residual r_i of agent i after the day-ahead market is given by*

$$\begin{aligned} q_i^{da,b} - q_i^{da,s} + \sum_{j\in\Gamma_i} q_{ij} &= d_i - F_i^{-1}\Big(\frac{p^{da,b} - p^{rt,s} - \mu_i^{da,b}}{p^{rt,b} - p^{rt,s}}\Big) \\ = d_i - F_i^{-1}\Big(\frac{p^{da,s} - p^{rt,s} + \mu_i^{da,s}}{p^{rt,b} - p^{rt,s}}\Big) &= d_i - F_i^{-1}\Big(\frac{c_{ij} - p^{rt,s} + \zeta_{ij} + \xi_{ij}}{p^{rt,b} - p^{rt,s}}\Big). \end{aligned} \tag{2}$$

More precisely,

$$r_i = F^{-1}\Big(\frac{p^{da,b} - p^{rt,s}}{p^{rt,b} - p^{rt,s}}\Big) \quad or \quad r_i = F^{-1}\Big(\frac{p^{da,s} - p^{rt,s}}{p^{rt,b} - p^{rt,s}}\Big) \tag{3}$$

if agent i purchases (or sells) electricity on the first stage.

The result above expresses the agents' decision on the day-ahead market in terms of residuals r_i, i.e. the quantities representing the additional purchases that each agent needs to make to balance the uncertainty of the supply after observing the realization of the renewable generation Δg_i. While not providing the explicit expressions for the decision variables $q_i^{da,b}, q_i^{s,da}, q_{ij}$, it is useful for further considerations. When designing market rules, it is important to choose an equilibrium with desirable properties from a set of equilibria (possibly infinite). In our analysis we rely on Generalized Nash Equilibria and its refinement, Variational Equilibria (VE) as discussed in Kulkarni and Shanbhag [2012].

Definition 1. *A Generalized Nash Equilibrium (GNE) of the game defined by the problem* (1) *with coupling constraints, is a vector* $x_i := (q_i^{da}, q_i^{rt}, \mathbf{q}_i)$ *that solves problem* (1) *or, equivalently, a vector* $x_i := (q_i^{da}, q_i^{rt}, \mathbf{q}_i)$ *such that* $x_i := (q_i^{da}, q_i^{rt}, \mathbf{q}_i)$ *solve the system* KKT_i *for each* i.

Definition 2. *A Variational Equilibrium (VE) of the game defined by the maximization problems* (1) *with coupling constraints, is a GNE of this game such that, in addition, the Lagrangian multipliers* ζ_{ij} *associated to the coupling constraints* $q_{ij} + q_{ji} = 0$ *are equal, i.e.:*

$$\zeta_{ij} = \zeta_{ji}, \quad \forall i \in \mathcal{N}, \forall j \in \Gamma_i \tag{4}$$

By duality theory, ζ_{ij} for $i \in \mathcal{N}, \forall j \in \Gamma_i$ can be interpreted as bilateral energy trading prices Le Cadre et al. [2020]. In general, ζ_{ij} might not be aligned with ζ_{ji}, thus leading to non-symmetric energy trading prices between couple of agents. Relying on VE as solution concepts enforces a natural symmetry in the bilateral energy price valuations Le Cadre et al. [2020]. The conditions on VE existence are proved in Shilov et al. [2023], where we also demostrate that the impact of the forecast update on the prosumer's outcome on the electricity market can be evaluated independently of the other prosumers' forecasts. Thus, it allows us to endogenize the utility of prosumers brought by the forecast update, which has been traditionally assumed as an exogenous factor in the literature on forecast models Raja et al. [2023], Lambert et al. [2008]. It was demonstrated that the efficiency of the VE of the electricity peer-to-peer market can be achieved if the prosumers participate in the forecast market, i.e. social optima can be achieved if the Market Operator has access to prosumers' forecasts.

Theorem 2 (Shilov et al. [2023]). *Total cost of agent* i *depends only on the parameters of agent* i*. It means that forecast market operator can compute utility change of agent* i *without information from the other agents.*

Theorem 3 (Shilov et al. [2023]). *If all the agents report their forecasts to the Market Operator (participate in the forecast market), then the VE of* (1) *coincides with the set of social welfare optima.*

While being a strong assumption, it is mitigated by the fact that we establish in the next section, more precisely, individual rationality of the coupling between forecast market and peer-to-peer electricity market. Comparing to Shilov et al. [2023], in which only the Gaussian distributions were considered, we show that under mild conditions for arbitrary distributions, agents benefit from purchasing the forecasts, thus, they have incentive for participation in the forecast market.

2.3 Individual Rationality

From (2) we obtain that $r_i = F_i^{-1}(\frac{\zeta_{ij}+c_{ij}-p^{rt,s}}{p^{rt,b}-p^{rt,s}})$, thus, bilateral trading cost is given by $\Pi_i^q = (\zeta_{ij} + c_{ij})\sum_{j\in\Gamma_i} q_{ij}$. From KKT conditions we have that $\zeta_{ij} + c_{ij} = c_i$ for each $j \in \Gamma_i$, where c_i is some constant specific for each agent with $p^{da,s} \le c_i \le p^{da,b}$. Then, from (2) we obtain that either c_i is equal to $p^{da,b}$, if agent i buys energy from the backup retailer, or to $p^{da,s}$ otherwise. It allows us to finally write expressions for the total cost imposed on the agent i. First, consider the case when i buys energy from backup retailer on the day-ahead market

$$\begin{aligned}\Pi_i^{total} &= p^{da,b}q_i^{da,b} + p^{da,b}\sum_{j\in\Gamma_i} q_{ij} + \Pi_i^{second} \\ &= p^{da,b}\Big[d_i - F_i^{-1}\Big(\frac{p^{da,b}-p^{rt,s}}{p^{rt,b}-p^{rt,s}}\Big)\Big] + \Pi_i^{second},\end{aligned} \tag{5}$$

where Π_i^{second} is given by

$$\begin{aligned}\Pi_i^{second} = &\overbrace{p^{rt,b}r_iF_r(r_i) + p^{rt,s}r_i(1-F_r(r_i))}^{1} \\ &\overbrace{-p^{rt,b}\int_0^{r_i}\Delta g_i f_r(\Delta g_i)d\Delta g_i - p^{rt,s}\int_{r_i}^{\infty}\Delta g_i f_r(\Delta g_i)d\Delta g_i}^{2},\end{aligned}$$

where F_r (f_r) denotes CDF (PDF) of a real distribution of Δg_i. It means that Π_i^{total} gives an expected cost of agent i who takes r_i as a first stage decision (r_i denotes residual after the first stage). Considering the first part of the expression:

$$p^{rt,b}r_iF_r(r_i) + p^{rt,s}r_i(1-F_r(r_i)) = r_i(p^{rt,b}-p^{rt,s})F_r(r_i) + p^{rt,s}r_i$$

The second part can be expressed as follows, where the expectation with respect to the real distribution is denoted as $\mathbb{E}_r[\cdot]$:

$$\begin{aligned}&p^{rt,b}\int_0^{r_i}\Delta g_i f_r(\Delta g_i)d\Delta g_i + p^{rt,s}\int_{r_i}^{\infty}\Delta g_i f_r(\Delta g_i)d\Delta g_i \\ &= p^{rt,b}\mathbb{E}_r[\Delta g_i|\Delta g_i \le r_i]\mathbb{P}_r(\Delta g_i \le r_i) + p^{rt,s}\mathbb{E}_r[\Delta g_i|\Delta g_i \ge r_i]\mathbb{P}_r(\Delta g_i \ge r_i) \\ &= p^{rt,b}\mathbb{E}_r[\Delta g_i|\Delta g_i \le r_i]F_r(r_i) + p^{rt,s}\mathbb{E}_r[\Delta g_i|\Delta g_i \ge r_i](1-F_r(r_i)) \\ &= p^{rt,b}\mathbb{E}_r[\Delta g_i] - (p^{rt,b}-p^{rt,s})\mathbb{E}_r[\Delta g_i|\Delta g_i \ge r_i](1-F_r(r_i))\end{aligned}$$

Summing it all together:

$$\begin{aligned}\Pi_i^{total} &= p^{da,b}[d_i - r_i] + r_i(p^{rt,b}-p^{rt,s})F_r(r_i) + p^{rt,s}r_i - p^{rt,b}\mathbb{E}_r[\Delta g_i] \\ &+ (p^{rt,b}-p^{rt,s})\mathbb{E}_r[\Delta g_i|\Delta g_i \ge r_i](1-F_r(r_i)),\end{aligned} \tag{6}$$

which gives us expected cost of an agent i.

Expression (6) allows us to consider the effects of the forecast market on the P2P market in an expectation with respect to the real distribution of Δg_i.

First, note that CDF $F_r(x)$ and its PDF $f_r(x)$ are defined on $x \in [0, \infty)$. The first question to answer (and to show the expected rewards of the sellers on the forecast market) is the definition of order between distributions. Naturally, we would like to show that $\mathbb{E}_r[\hat{\Pi}_i^{total}] \leq \mathbb{E}_r[\Pi_i^{total}]$ if distribution $\hat{F}_i$ is "better" than F_i. Intuitively, for one shot game it should compare two distributions by the amount of probability mass concentrated around the realization of a random variable Δg_i. It provide us a hint that the comparison should be made by conditioning the distance between distributions. The question on how to choose the metrics is non-trivial as shown below.

Consider an agent i who has an initial forecast about the distribution of Δg_i with CDF F_i and a 'better' forecast with PDF $\hat{F}_i$. Then, we want to show that

$$\mathbb{E}_r[\hat{\Pi}_i^{total}] \leq \mathbb{E}_r[\Pi_i^{total}] \tag{7}$$

Now, fix prices $p^{rt,b} > p^{da,b} > p^{da,s} > p^{rt,s}$ and denote $\rho := \frac{p^{da,b} - p^{rt,s}}{p^{rt,b} - p^{rt,s}}$. Assume that agent i buys energy on the day-ahead market if she uses F_i or $\hat{F}_i$ or F_r (this can be expressed as $F_i^{-1}(\rho), \hat{F}_i^{-1}(\rho), F_r^{-1}(\rho) \leq d_i$). Situation in which i sells energy on the day-ahead market is considered similarly. Moreover, denote $r_i^r := F_r^{-1}(\rho)$ and $\hat{r}_i := \hat{F}_i^{-1}(\rho)$.

Denote as Π_r^{total} the cost obtained by the agent i associated with the decision $r_i = F_r^{-1}(\rho)$, taken when she knows the real distribution F_r. Subtracting it from both sides of (7), and using (6) we write for the right side of the inequality

$$\begin{aligned} S_r := &\, p^{da,b}[r_i^r - r_i] + p^{rt,s}[r_i - r_i^r] + (p^{rt,b} - p^{rt,s})[r_i F_r(r_i) - r_i^r F_r(r_i^r)] \\ &+ (p^{rt,b} - p^{rt,s})\Big[\int_{r_i}^{\infty} x f_r(x)dx - \int_{r_i^r}^{\infty} x f_r(x)dx\Big] \end{aligned}$$

with the left side (S_l) written in the same way but with $\hat{r}_i$ instead of r_i. Now, dividing both sides by $(p^{rt,b} - p^{rt,s})$, we can write S_r (or S_l if we use $\hat{F}_i$) as

$$\begin{aligned} S_r &= \rho[F_r^{-1}(\rho) - F_i^{-1}(\rho)] + [F_i^{-1}(\rho)F_r(F_i^{-1}(\rho)) - F_r^{-1}(\rho)F_r(F_r^{-1}(\rho))] \\ &\quad + \Big[\int_{F_i^{-1}(\rho)}^{\infty} x f_r(x)dx - \int_{F_r^{-1}(\rho)}^{\infty} x f_r(x)dx\Big] \\ &= F_i^{-1}(\rho)[F_r(F_i^{-1}(\rho)) - \rho] + \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} x f_r(x)dx \end{aligned}$$

Integrating by parts gives

$$\begin{aligned} S_r &= F_i^{-1}(\rho)[F_r(F_i^{-1}(\rho)) - \rho] + F_r^{-1}(\rho)F_r(F_r^{-1}(\rho)) \\ &\quad - F_i^{-1}(\rho)F_r(F_i^{-1}(\rho)) - \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} F_r(x)dx = \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} (\rho - F_r(x))dx \end{aligned}$$

Thus, we want to prove that

$$\int_{\hat{F}_i^{-1}(\rho)}^{F_r^{-1}(\rho)} (\rho - F_r(x))dx \leq \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} (\rho - F_r(x))dx \tag{8}$$

Assume now that for given ρ,

$$|\hat{F}_i^{-1}(\rho) - F_r^{-1}(\rho)| \leq |F_i^{-1}(\rho) - F_r^{-1}(\rho)| \tag{9}$$

which does not immediately guarantee that (8) holds without additional assumptions on F_r in the neighbourhood of $F_r^{-1}(\rho)$. Next we discuss the conditions on the distributions and ρ such that (7) holds. First, note that $\rho = F_r(F_r^{-1}(\rho))$, thus, inequality clearly holds when $F_i^{-1}(\rho) \leq \hat{F}_i^{-1}(\rho) \leq F_r^{-1}(\rho)$ or when $F_i^{-1}(\rho) \geq \hat{F}_i^{-1}(\rho) \geq F_r^{-1}(\rho)$. We next assume that $\hat{F}_i^{-1}(\rho) \leq F_r^{-1}(\rho) \leq F_i^{-1}(\rho)$, while the opposite case can be considered similarly. In this case with the change of variables we can rewrite (8) as

$$\int_{F_r(\hat{F}_i^{-1}(\rho))}^{\rho} [F_r^{-1}(x) - \hat{F}_i^{-1}(\rho)]dx \leq \int_{\rho}^{F_r(F_i^{-1}(\rho))} [F_i^{-1}(\rho) - F_r^{-1}(x)]dx \tag{10}$$

in which the left part is upper-bounded by $\int_{F_r(\hat{F}_i^{-1}(\rho))}^{\rho} [F_r^{-1}(\rho) - \hat{F}_i^{-1}(\rho)]$ and the left part is lower-bounded by $\int_{\rho}^{F_r(F_i^{-1}(\rho))} [F_i^{-1}(\rho) - F_r^{-1}(\rho)]$.

Theorem 4. *Forecast's update from F_i to $\hat{F}_i$ decreases agent i's costs (i.e. inequality (7) holds) if*

1. $|\hat{F}_i^{-1}(\rho) - F_r^{-1}(\rho)| \leq |F_i^{-1}(\rho) - F_r^{-1}(\rho)|$
2. $\frac{b-a}{c-a} \int_a^c f_r(x)dx \leq \int_b^c f_r(x)dx$,.

where $a := \hat{F}_i^{-1}(\rho), b := F_r^{-1}(\rho), c := F_i^{-1}(\rho)$ *and* F_r *denotes real CDF of* Δg_i.

Proof. Proof follows from the derivations above. Using the bounds in (10) and denoting $a := \hat{F}_i^{-1}(\rho), b := F_r^{-1}(\rho), c := F_i^{-1}(\rho)$ we can write it as

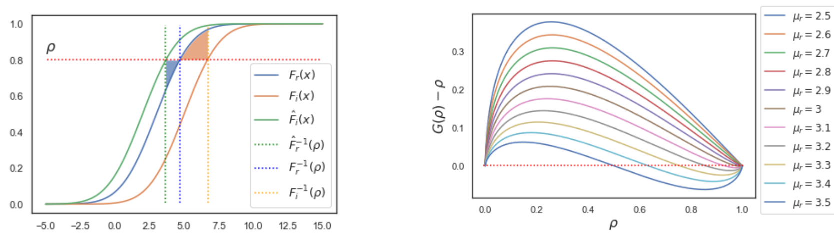
$$(b-a)[F_r(b) - F_r(a)] \leq (c-b)[F_r(c) - F_r(b)] \tag{11}$$

Which can then be transformed into

$$\frac{b-a}{c-a} \int_a^c f_r(x)dx \leq \int_b^c f_r(x)dx \tag{12}$$

where coefficient $K := \frac{b-a}{c-a} \in (0, \frac{1}{2}]$. This, combined with (9) gives exactly the conditions of the theorem. Note that the case with $F_i^{-1}(\rho) \leq F_r^{-1}(\rho) \leq \hat{F}_i^{-1}(\rho)$ is considered similarly. □

Taking the worst case with $K = \frac{1}{2}$, we obtain that the condition for (12) holds if $F_r''(x) \geq 0$ for $x \in [a, c]$ which represents a sufficient condition for (7) to hold. In the general case, condition (12) defines the relationship between the quantiles of the forecasts and CDF of a real distirbution of Δg_i. This condition is illustrated in Figure 2a: blue area should be less than the orange one.



(a) Condition (12). Blue area should be smaller than the orange one.

(b) $G(\rho) - \rho$ for various distances between distributions.

Fig. 2: Conditions for general and Gaussian distirbution

Example 1. To illustrate the implications of our result we consider the following example: let F_r, F_i and $\hat{F}_i$ represent CDFs of Gaussian distributions with means μ_r, μ_i and $\hat{\mu}_i$ respectively, where $\hat{\mu}_i \leq \mu_r \leq \mu_i$ and $\mu_r - \hat{\mu}_i \leq \mu_i - \mu_r$. Assume that the variance is the same for all the distributions. For such shifted Gaussian distributions, condition (12) reduces to the following upper bound:

$$\rho \leq G(\rho) := \frac{\mu_i - \mu_r}{\mu_i - \hat{\mu}_i} \Phi\big(\frac{\mu_i - \mu_r}{\sigma} + \Phi^{-1}(\rho)\big) - \frac{\hat{\mu}_i - \mu_r}{\mu_i - \hat{\mu}_i} \Phi\big(\frac{\hat{\mu}_i - \mu_r}{\sigma} + \Phi^{-1}(\rho)\big),$$

which can be easily evaluated numerically. Figure 2b demonstrates the values of $G(\rho) - \rho$ for different μ_r while μ_i and $\hat{\mu}_i$ are fixed and are equal to 5 and 2 respectively. The closer $\hat{\mu}_i$ to μ_r comparing to $\mu_i - \mu_r$, the bigger admissible values of ρ are. For example, when $\mu_r = 3$ (as in Figure 2a), condition (12) is satisfied with $\rho \lesssim 0.97453$. As demonstrated in Shilov et al. [2023], it is possible to obtain tighter bounds with certain conditions on the distributions, while theorem 4 provides conditions for arbitrary pdfs.

3 Conclusion

In this work, we formulated a coupling model between a P2P market and a forecast market. We addressed existence of incentives for the prosumers to participate in the forecast market. In addition, we proved the conditions on the 'distance' between the distributions purchase of the forecast leads to decreased costs. This is a major result which highlights that it is profitable for the prosumers to purchase forecasts and that leads to reaching a social optimum of the peer-to-peer market.

This paper contributes to a novel direction of exploring the connection between electricity and forecast markets. One of the promising directions for further research is to apply the model to real-time markets involving dynamic prices and uncertainty coming from high share of renewable generation. Continuing in these directions, we can move towards the most efficient way for the forecast markets to interact with electricity markets with renewable generation.

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