Tariff Versus Sanction Under Bounded Rationality*

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Abstract. We formulate the design of a taxation mechanism as a Stackelberg game assuming: a) perfect competition, with exogenous prices; b) imperfect competition, captured through a variational inequality approach, with endogenous prices. Three settings of the mechanism are considered: (i) benchmark involving no taxation, (ii) optimum tariff, (iii) optimum sanction. The expected utility maximization formulation of the game is extended further by relying on cumulative prospect theory to account for the bounded rationality of the stakeholders. We derive closed-form mappings linking the outcomes of the three settings. Additionally, we assess the impact of bounded rationality through a new performance metric, the Price of Irrationality. Numerical results are derived on a randomized instance of a gas trading game between Europe, Asia, and Russia.

Keywords: Game Theory · Sanction Design · Cumulative Prospect Theory.

1 Introduction

Problem Statement: The economies of many geographic markets are dependent on fossil fuels, e.g., gas. Due to geopolitical tensions, some geographic markets may decide to partially or totally halt their gas imports by imposing tariffs/sanctions on the imports [5]. We therefore propose a model which casts the optimal tariff/sanction problem as a Stackelberg game in two settings: a) perfect competition involving price-taking agents, i.e., geographic markets without market power, and a Global Market Operator (GMO) defining exogenous export prices; b) imperfect competition where export prices are determined endogenously as a result of the interactions among geographic markets. The model is extended further to incorporate the bounded rationality of the agents, relying on Prospect Theory (PT).

Main Contributions: Our model is built on an agent-based representation of suppliers and generators interacting in a certain number of geographic markets. We aim to assess the impact of tariffs/sanctions on the geographic markets' imports of gas and on their welfare, framing the optimal sanction model as a Stackelberg game. Considering the bounded rationality of the stakeholders with respect to a potential risk of supply shortage, we rely on PT to extend the optimal taxation/sanction games to situations involving stakeholders with different risk-aversion levels. Finally, numerical results are derived on a randomized instance of the problem.

Reviewed Literature: We consider two forms of taxation mechanisms: tariff and sanction. The optimum tariff is determined by a regulator, interpreted as the leader of a Stackelberg game, to maximize the welfare of the geographic market that imposes the tax, while anticipating the clearing of the global gas exchange market. Differently, the optimum sanction is determined by a regulator that arbitrages between maximizing the welfare of the geographic market which imposes the tax, and minimizing the welfare of the geographic market to be sanctioned, while anticipating the clearing of the global gas exchange market. Although they are simple economic instruments designed to inflict economic harm, they have wider implications and can be used

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to exert pressure to change behavior or policies. However, the literature dealing primarily with the optimal design of sanctions is scarce. For this reason, our main bibliographic reference is [5]. Specifically, [5] applies different sanction models to the specific context of Europe's dependence on Russian gas by framing the problem as a two-player partial equilibrium model and it assesses the impact of these sanctions on the utilities of both stakeholders. We generalize this work in several directions. First, we formulate taxation models as Stackelberg games under perfect and imperfect competition, considering a larger number of stakeholders. Second, the impact of the bounded rationality of the stakeholders on the market equilibria is considered relying on the new concept of Price of Irrationality (PoI).

Paper Organization: The paper is organized as follows. In Section 3, we formulate the perfect competition setting as a partial equilibrium model for three settings (i) benchmark, (ii) optimum tariff, (iii) optimum sanction. The model is extended further relying on PT to capture the stakeholders' bounded rationality in Section 4. The Price of Irrationality is introduced in Section 4.2 as a performance metric to assess the efficiency loss caused by the stakeholders' bounded rationality in each setting. We conclude in Section 5.

2 The Geographic Markets

Let $\mathcal{N}$ be a set of N geographic Natural Gas (NG) markets, abstracted as nodes on a directed graph $\Gamma = (\mathcal{N}, E)$ where $E \subseteq \mathcal{N} \times \mathcal{N}$ is the set of oriented edges between the agents. Our model is built on an agent-based representation of suppliers and generators interacting in a certain number of geographic markets. Markets can be either *demand markets* with no generation facility, or *generation markets* with fixed exogenously defined demands. After satisfying their own demand, generation markets can sell their generation surplus to the demand markets. Let $\mathcal{N}^d$ and $\mathcal{N}^g$ be the sets of demand and generation markets resp., such that $\mathcal{N} = \mathcal{N}^d \sqcup \mathcal{N}^g$, $\mathcal{N}^d \sqcap \mathcal{N}^g = \emptyset$. For a given market $n \in \mathcal{N}$, we define as $\mathcal{N}_n$ the subset of markets it can import (resp. export) NG from (resp. to). Let $d_n \in \mathbb{R}_{\geq 0}$ be the demand of market n, $q_n \in \mathbb{R}_{\geq 0}$ its generation, and $q_{m,n}, \forall m \in \mathcal{N}_n$ the amount that market n imports (resp. exports) from (resp. to) the other markets. We impose the following convention: $q_{m,n} \geq 0$ means that n buys NG from market m; while $q_{m,n} \leq 0$ means that n sells NG to market n. In this setting, each market $n \in \mathcal{N}$ is described by a strategy vector $\boldsymbol{x}_n \overset{\text{def}}{=} (d_n, q_n, (q_{m,n})_{m \in \mathcal{N}_n}) \in \mathbb{R}^{m_n}$ with $m_n = |\mathcal{N}_n| + 2$. We let $\boldsymbol{x}_{-n} \overset{\text{def}}{=} (\boldsymbol{x}_m)_{m \neq n}$ be the vector which contains the decisions of all the agents in $\mathcal{N}$ except n. Furthermore, we define $\boldsymbol{x} \overset{\text{def}}{=} \text{col}\left((\boldsymbol{x}_n)_{n \in \mathcal{N}}\right)$ as the collection of the N players' decision variables, and $\boldsymbol{x}_0 \overset{\text{def}}{=} (p_{n,m})_{n,m}$ as the vector containing the trading prices between each couple of geographic markets in interaction. For illustrative purpose, we consider 3 geographic markets: the European Union (EU), Asia (A) and Russia (R). Generalization to a larger number of markets is straightforward.

Assumption 1 *EU is a demand market with no NG production facilities, thus* $q_{EU} = 0$*; A and R are generation markets with both fixed demands, i.e.,* $d_R = \bar{d}_R > 0$ *and* $d_A = \bar{d}_A > 0$.

Assumption 2 *[10] In the geographic demand market* $n \in \mathcal{N}^d$*,* d_n *is assumed to be a linearly decreasing function of the price paid by its consumers (domestic NG price)* p_n*, i.e.,* $d_n(p_n) = \alpha_n - \beta_n p_n \Leftrightarrow p_n(d_n) = \frac{\alpha_n - d_n}{\beta_n}$ *with* $\alpha_n \geq 0, \beta_n > 0$.

From Asm. 1, $\mathcal{N}^d \overset{\text{def}}{=} \{EU\}$, $\mathcal{N}^g \overset{\text{def}}{=} \{A, R\}$. We can now define the stakeholders:

- Demand Market: In our case, it is composed solely of the EU with demand d_{EU}, which is modeled as a linearly decreasing function of p_{EU} in line with Asm. 2. The EU's utility function, interpreted as its welfare, is defined as the difference between the consumer surplus and the trades costs: $J_{\text{EU}}(\boldsymbol{x}_{\text{EU}}, \boldsymbol{x}_0) = \frac{1}{\beta_{\text{EU}}}\left(\alpha_{\text{EU}} d_{\text{EU}} - \frac{d_{\text{EU}}^2}{2}\right) - p_{\text{EU,R}} q_{\text{R,EU}} - p_{\text{EU,A}} q_{\text{A,EU}}$.

- Generation Markets: Asia (A) and Russia (R) are the two generation markets whose utility functions are defined as the difference between revenues from trades and gas generation costs. Following [10], the gas generation cost is assumed to be quadratic in q_n, i.e., $C_n(q_n) = a_n q_n^2 + b_n q_n + c_n$, $\forall n \in \mathcal{N}^g$ with $a_n > 0, b_n \geq 0, c_n \geq 0$. We have: $J_n(\boldsymbol{x}_n, \boldsymbol{x}_0) = \sum_{m \in \mathcal{N}_n} p_{n,m} q_{m,n} - C_n(q_n), \quad \forall n \in \mathcal{N}^g$.
- Global Market Operator (GMO): The GMO aims to minimize the trade surplus. Its utility function is formally given by $J_0(\boldsymbol{x}, \boldsymbol{x}_0) = \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{N}_n} [p_{n,m} q_{m,n} + p_{m,n} q_{n,m}]$. The choice of $J_0(\cdot)$ is classical in in partial equilibrium models and essentially justified by the willingness to enforce global market clearing. Let $Q_n \stackrel{\text{def}}{=} \sum_{m \in \mathcal{N}_n} q_{m,n}$ be market n's net import for NG.

  Denote $\mathcal{X}_n \stackrel{\text{def}}{=} \{\boldsymbol{x}_n \in \mathbb{R}^{m_n} \mid d_n \leq q_n + Q_n, q_n = 0 \vee d_n = \bar{d}_n\}$ as the feasibility set of market $n \in \mathcal{N}$, i.e., the set of decision variables satisfying the local balance of supply and demand, and $\mathcal{X} \stackrel{\text{def}}{=} \prod_{n \in \mathcal{N}} \mathcal{X}_n$ as the markets' joint feasibility set. Similarly, we let $\mathcal{X}_0$ be the feasibility set of the GMO.

3 Market Structures

3.1 Price-Taker Formulation

The perfect competition setting can be formulated as a partial equilibrium model [1,6], which is a simplified interpretation of the general equilibrium theory [1]. In this setting, the GMO, interpreted as an external agent, acts as the "invisible hand" of Adam Smith, setting the trade prices. Let $\mathcal{G} = (\mathcal{N} \sqcup \{\text{GMO}\}, \mathcal{X} \times \mathcal{X}_0, (J_n)_n)$. The geographic markets solve the following non-cooperative game to determine their optimal strategy:

$$\forall n \in \mathcal{N}, \max_{\boldsymbol{x}_n} \quad J_n(\boldsymbol{x}_n, \boldsymbol{x}_0), \qquad \max_{\boldsymbol{x}_0} \quad J_0(\boldsymbol{x}, \boldsymbol{x}_0), \tag{1a}$$

$$\text{s.t.} \quad \boldsymbol{x}_n \in \mathcal{X}_n, \qquad \text{s.t.} \quad 0 \leq p_{n,m} \leq \bar{p}, \forall n \in \mathcal{N}, \forall m \in \Gamma_n, \tag{1b}$$

$$p_{n,m} = p_{m,n}, \forall n \in \mathcal{N}, \forall m \in \Gamma_n. \tag{1c}$$

Simultaneously, the GMO sets the export prices, given $\boldsymbol{x}_n$, $\forall n$. In (1), we impose that the trade prices, chosen by the price-setting agent, are limited by the market's price cap $\bar{p} > 0$ and price floors. We assume a lower limit of 0 for all trade prices. Import and export trading price symmetry is captured by (1c). The partial equilibrium model outcome can be interpreted as a Nash Equilibrium (NE), defined as follows:

Definition 1 *[1], [2] A strategy profile $\widetilde{\boldsymbol{x}}^* = (\boldsymbol{x}^*, \boldsymbol{x}_0^*) \in \mathcal{X} \times \mathcal{X}_0$ is a Nash Equilibrium (NE) solution of $\mathcal{G}$ if, and only if, $J_n(\widetilde{\boldsymbol{x}}_n^*, \widetilde{\boldsymbol{x}}_{-n}^*) \geq J_n(\widetilde{\boldsymbol{x}}_n, \widetilde{\boldsymbol{x}}_{-n}^*), \forall \widetilde{\boldsymbol{x}}_n \in \mathcal{X}_n, \forall n \in \mathcal{N} \sqcup \{GMO\}$.*

We note that at a Nash Equilibrium, $J_0^* = J_0(\boldsymbol{x}^*, \boldsymbol{x}_0^*) = 0$ from (1) and $J_0(\cdot)$ definition.

Proposition 1 *There exists a NE solution of the Nash Game $\mathcal{G}$.*

Proof. The statement follows from [8] observing that $J_n(\cdot), \forall n \in \mathcal{N} \sqcup \{\text{GMO}\}$ are concave functions and the joint feasibility set $\mathcal{X} \times \mathcal{X}_0$ is convex and compact as the intersection of affine constraints. □

Proposition 2 *At a NE, $d_n^* = q_n^* + Q_n^*, \forall n \in \mathcal{N}$.*

Proof. Reasoning by contradiction, we assume that at equilibrium there exists $n \in \mathcal{N}$ such that $d_n^* < q_n^* + Q_n^*$. As a result, for this $n \in \mathcal{N}$, $\lambda_n^* = 0$. But, from the KKTs and the constraint on bilateral trades price symmetry, this implies that $\lambda_n^* = 0, \forall n \in \mathcal{N}$. As a result, R and A make no profit from exports. Then, in order to maximize its profits function, while satisfying the demand constraints, the generation variables will take the values $\overline{d}_{\text{R}}$ and $\overline{d}_{\text{A}}$, respectively.

Therefore $Q^*_{\text{R}} = Q^*_{\text{A}} = 0$. This implies in turn that $q^*_{\text{R,A}} = q^*_{\text{R,EU}} = q^*_{\text{A,EU}} = 0$, and then $Q^*_{\text{EU}} = 0$. Then, from the primal feasibility constraint for EU we get that $d^*_{\text{EU}} \leq 0$, which is in contradiction with the form of the EU's utility function which is quadratic in d_{EU} with a maximum on $\mathbb{R}^*_+$. The proposition statement follows. □

Proposition 3 *There exist multiple NEs solutions of the game $\mathcal{G}$.*

Proof. Let $\lambda_n \geq 0, \forall n$, be the dual variables associated with the supply-demand balance inequalities. At equilibrium, from the stationarity conditions, we get that $p^*_{\text{EU,A}} = \lambda^*_{\text{EU}}$, $p^*_{\text{EU,R}} = \lambda^*_{\text{EU}}$, $p^*_{\text{A,EU}} = \lambda^*_{\text{A}}$, $p^*_{\text{A,R}} = \lambda^*_{\text{A}}$, $p^*_{\text{R,EU}} = \lambda^*_{\text{R}}$, $p^*_{\text{R,A}} = \lambda^*_{\text{R}}$. From this set of relationships, we infer that $p^*_{\text{EU,A}} = p^*_{\text{EU,R}}$, $p^*_{\text{A,EU}} = p^*_{\text{A,R}}$, $p^*_{\text{R,EU}} = p^*_{\text{R,A}}$ and $\lambda^*_{\text{EU}} = \lambda^*_{\text{R}} = \lambda^*_{\text{A}} = \lambda^*$. In addition, we have the following expressions for demand and generations at NE: $d^*_{\text{EU}} = \alpha_{\text{EU}} - \beta_{\text{EU}}\lambda^*_{\text{EU}}$, $q^*_A = \frac{1}{2a_{\text{A}}}(\lambda^*_{\text{A}} - b_{\text{A}})$ and $q^*_R = \frac{1}{2a_{\text{R}}}(\lambda^*_{\text{R}} - b_{\text{R}})$.
From Prop. 2, we have:

$$Q^*_{\text{EU}} = d^*_{\text{EU}},\ Q^*_{\text{R}} = \bar{d}_{\text{R}} - q^*_{\text{R}},\ Q^*_{\text{A}} = \bar{d}_{\text{A}} - q^*_{\text{A}}. \tag{2}$$

Furthermore, the bilateral trading reciprocity constraint implies that $\sum_n Q^*_n = 0$. This enables us to prove the existence of a unique market price:

$$\lambda^* = \frac{\alpha_{\text{EU}} + \left[\bar{d}_{\text{A}} + \frac{b_{\text{A}}}{2a_{\text{A}}} + \bar{d}_{\text{R}} + \frac{b_{\text{R}}}{2a_{\text{R}}}\right]}{\beta_{\text{EU}} + \frac{b_{\text{A}}}{2a_{\text{A}}} + \frac{b_{\text{R}}}{2a_{\text{R}}}} \tag{3}$$

Regarding the trades, from Equation 2, we obtain a system of three equations with three unknown variables:

$$\begin{cases} q_{\text{R,EU}} + q_{\text{A,EU}} = d^*_{\text{EU}}, \\ -q_{\text{R,EU}} + q_{\text{A,R}} = \bar{d}_{\text{R}} - q^*_{\text{R}}, \\ -q_{\text{A,EU}} - q_{\text{A,R}} = \bar{d}_{\text{R}} - q^*_{\text{A}}. \end{cases} \tag{4}$$

It is easy to see that this resulting system is incompatible whenever $d^*_{\text{EU}} + \bar{d}_{\text{A}} - q^*_A + \bar{d}_{\text{R}} - q^*_{\text{R}} \neq 0$, which never holds due to the construction of the game. Then, $d^*_{\text{EU}} + \bar{d}_A - q^*_A + \bar{d}_{\text{R}} - q^*_{\text{R}} = 0$ always and the system is linearly dependent. Thus, the NE is not uniquely defined. □

Let SOL($\mathcal{G}$) denote the set of solutions of the game $\mathcal{G}$.

Proposition 4 $\forall n \in \mathcal{N} \sqcup \{GMO\}, J_n(\widetilde{\boldsymbol{x}}^*) = J_n(\widetilde{\boldsymbol{x}}^{**}), \forall \widetilde{\boldsymbol{x}}^*, \widetilde{\boldsymbol{x}}^{**} \in SOL(\mathcal{G})$.

Proof. By substitution of the optimal net imports in the markets' utility functions, at each NE we obtain the following expressions: $J^*_{\text{EU}} = \frac{\alpha^2_{\text{EU}}}{2\beta_{\text{EU}}} - \alpha_{\text{EU}}\lambda^* + \frac{1}{2}\beta_{\text{EU}}(\lambda^*)^2$, $J^*_n = \left[\frac{b^2_n}{4a_n} - c_n\right] - \left[\bar{d}_n + \frac{b_n}{2a_n}\right]\lambda^* + \frac{(\lambda^*)^2}{4a_n}, \quad \forall n \in \{A, R\}$ which depend only on $\mathcal{G}$ parameters. □

3.2 Price-Maker Formulation

In a decentralized market, geographic markets' decisions influence the market prices which are obtained as dual variables of the bilateral trading reciprocity equations. In this setting, the geographic markets' utility functions are modified as follows: $J^v_{\text{EU}}(\boldsymbol{x}_{\text{EU}}) = \frac{1}{\beta_{\text{EU}}}\left(\alpha_{\text{EU}} d_{\text{EU}} - \frac{d^2_{\text{EU}}}{2}\right)$ and $J^v_{\text{n}}(\boldsymbol{x}_{\text{n}}) = -(a_{\text{n}} q^2_{\text{n}} + b_{\text{n}} q_{\text{n}} + c_{\text{n}}), \forall n \in \{A, R\}$ and the feasibility sets become coupled, i.e., $\mathcal{X}_n(\boldsymbol{x}_{-n}) = \{\boldsymbol{x}_n \in \mathcal{X}_n \,|\, q_{n,m} = -q_{m,n},\, p_{n,m} = p_{m,n},\, \forall m \in \mathcal{N}_n\}$, $\forall n$. Thus, each agent $n \in \mathcal{N}$ solves the following optimization problem:

$$\begin{aligned} \max_{\boldsymbol{x}_{\text{n}}} \quad & J^v_n(\boldsymbol{x}_{\text{n}}), \\ \text{s.t.} \quad & \boldsymbol{x}_n \in \mathcal{X}_n(\boldsymbol{x}_{-n}). \end{aligned} \tag{5}$$

The Variational Equilibria (VEs) solutions of the resulting game can be interpreted as NEs of the game $\mathcal{G}^{\text{v}} = (\mathcal{N}, (\mathcal{X}_n(\boldsymbol{x}_{-n}))_n, (J^v_n)_n)$.

Proposition 5 *There exist multiple VEs solutions of $\mathcal{G}^{v}$.*

Proof. At equilibrium, from the stationarity conditions, we get that $\lambda^*_{\mathrm{EU,A}} = \lambda^*_{\mathrm{EU}}$, $\lambda^*_{\mathrm{EU,R}} = \lambda^*_{\mathrm{EU}}$, $\lambda^*_{\mathrm{A,EU}} = \lambda^*_{\mathrm{A}}$, $\lambda^*_{\mathrm{A,R}} = \lambda^*_{\mathrm{A}}$, $\lambda^*_{\mathrm{R,EU}} = \lambda^*_{\mathrm{R}}$, $\lambda^*_{\mathrm{R,A}} = \lambda^*_{\mathrm{R}}$. From this set of relationships, we infer that $\lambda^*_{\mathrm{EU,A}} = \lambda^*_{\mathrm{EU,R}}$, $\lambda^*_{\mathrm{A,EU}} = \lambda^*_{\mathrm{A,R}}$, $\lambda^*_{\mathrm{R,EU}} = \lambda^*_{\mathrm{R,A}}$ and $\lambda^*_{\mathrm{EU}} = \lambda^*_{\mathrm{R}} = \lambda^*_{\mathrm{A}} = \lambda^*$.

In addition, we have the following expressions for demand and generations at a VE: $d^*_{\mathrm{EU}} = \alpha_{\mathrm{EU}} - \beta_{\mathrm{EU}}\lambda^*_{\mathrm{EU}}$, $q^*_{\mathrm{A}} = \frac{1}{2a_{\mathrm{A}}}(\lambda^*_{\mathrm{A}} - b_{\mathrm{A}})$ and $q^*_{\mathrm{R}} = \frac{1}{2a_{\mathrm{R}}}(\lambda^*_{\mathrm{R}} - b_{\mathrm{R}})$.

The net imports also give us: $Q^*_{\mathrm{EU}} = d^*_{\mathrm{EU}}$, $Q^*_{\mathrm{R}} = \bar{d}_{\mathrm{R}} - q^*_{\mathrm{R}}$, $Q^*_{\mathrm{A}} = \bar{d}_{\mathrm{A}} - q^*_{\mathrm{A}}$

But, the bilateral trading reciprocity constraint implies that $\sum_n Q^*_n = 0$. This enables us to prove the existence of an interior point NE in λ^* defined in (3).

□

Let SOL($\mathcal{G}^{v}$) denote the set of VEs solutions of $\mathcal{G}^{v}$. Similarly to Proposition 4, we prove that the markets' utility are constant over SOL($\mathcal{G}^{v}$). Substituting $(Q^*_n)_n$ in the markets' utility functions, at each VE we obtain the following expressions: $J^{v^*}_{\mathrm{EU}} = -\frac{\beta_{\mathrm{EU}}}{2}\lambda^* + \frac{\alpha^2_{\mathrm{EU}}}{2\beta_{\mathrm{EU}}}$, $J^{v^*}_{\mathrm{n}} = -\frac{1}{4a_{\mathrm{n}}}\lambda^* + \frac{b^2_{\mathrm{n}}}{4a_{\mathrm{n}}} - c_{\mathrm{n}}, \quad \forall n \in \{A, R\}$ which depend only on $\mathcal{G}^{v}$ parameters.

3.3 Optimum Tariff

In the price-taker model defined in Section 3.1, a tariff (τ) is imposed by the GMO on imports from Russia, leading to modified utilities for the geographic markets:

$$\begin{aligned}
J_{\mathrm{EU}}(\boldsymbol{x}_{\mathrm{EU}}, \boldsymbol{x}_0, \tau) &= \frac{1}{\beta_{\mathrm{EU}}}\left(\alpha_{\mathrm{EU}} d_{\mathrm{EU}} - \frac{d^2_{\mathrm{EU}}}{2}\right) - (p_{\mathrm{EU,R}} - \tau) q_{\mathrm{R,EU}} - p_{\mathrm{EU,A}} q_{\mathrm{A,EU}},\\
J_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{x}_0, \tau) &= p_{\mathrm{A,R}} q_{\mathrm{A,R}} + p_{\mathrm{A,EU}} q_{\mathrm{A,EU}} - C_{\mathrm{A}}(q_{\mathrm{A}}),\\
J_{\mathrm{R}}(\boldsymbol{x}_{\mathrm{R}}, \boldsymbol{x}_0, \tau) &= p_{\mathrm{R,A}} q_{\mathrm{R,A}} + (p_{\mathrm{R,EU}} - \tau) q_{\mathrm{R,EU}} + p_{\mathrm{A,R}} q_{\mathrm{A,R}} - C_{\mathrm{R}}(q_{\mathrm{R}}).
\end{aligned}$$

The tariff τ being fixed, let $\mathcal{G}_\tau = (\mathcal{N} \sqcup \mathcal{N}_0, \mathcal{X} \times \mathcal{X}_0, (J_n(\cdot, \tau))_n)$ and SOL($\mathcal{G}_\tau$) denote the set of solutions of $\mathcal{G}_\tau$. The optimum tariff problem can be formulated as a single leader-multiple followers Stackelberg game [2]:

$$\begin{aligned}
\max_{\tau, \boldsymbol{x}} \quad & J_{\mathrm{EU}}(\boldsymbol{x}_{\mathrm{EU}}, \tau),\\
\text{s.t.} \quad & \forall n \in \mathcal{N}, \boldsymbol{x}_n \in \mathrm{SOL}(\mathcal{G}_\tau).
\end{aligned} \tag{6}$$

Proposition 6 *The market price in $\mathcal{G}_\tau$ can be expressed as an affine function of the market price in $\mathcal{G}$, $\lambda^t(\tau) = \lambda^* + \frac{\beta_{EU}\,\tau}{\beta_{EU} + \frac{1}{2\,a_A} + \frac{1}{2\,a_R}}$. Furthermore, there exists a unique tariff τ^* solution of the Stackelberg game* (6)*.*

Proof. For fixed τ, relying on the stationarity conditions of the lower level of (6), from the trades price symmetry, we infer that $\lambda^*_{\mathrm{EU}}(\tau) = \lambda^t(\tau) - \tau$, $\lambda^*_{\mathrm{A}}(\tau) = \lambda^t(\tau)$, $\lambda^*_{\mathrm{R}}(\tau) = \lambda^t(\tau)$.
In addition, we have the following expressions for demand and generations at NE of the lower level: $d^*_{\mathrm{EU}}(\tau) = \alpha_{\mathrm{EU}} - \beta_{\mathrm{EU}}\lambda^*_{\mathrm{EU}}(\tau)$, $q^*_{\mathrm{A}}(\tau) = \frac{\lambda^*_{\mathrm{A}}(\tau) - b_{\mathrm{A}}}{2a_{\mathrm{A}}}$, and $q^*_{\mathrm{R}}(\tau) = \frac{\lambda^*_{\mathrm{R}}(\tau) - \tau - b_{\mathrm{R}}}{2a_{\mathrm{R}}}$.
From Prop. 2, we have $Q^*_{\mathrm{EU}}(\tau) = d^*_{\mathrm{EU}}(\tau), Q^*_{\mathrm{R}}(\tau) = \bar{d}_{\mathrm{R}} - q^*_{\mathrm{R}}(\tau), Q^*_{\mathrm{A}}(\tau) = \bar{d}_{\mathrm{A}} - q^*_{\mathrm{A}}(\tau)$. Furthermore, the bilateral trading reciprocity constraint implies that $\sum_n Q^*_{\mathrm{n}}(\tau) = 0$. This enables us to prove the existence of an interior point NE of the lower level:

$$\lambda^t(\tau) = \frac{\alpha_{\mathrm{EU}} + \bar{d}_{\mathrm{A}} + \bar{d}_{\mathrm{R}} + \frac{b_{\mathrm{A}}}{2\,a_{\mathrm{A}}} + \frac{b_{\mathrm{R}}}{2\,a_{\mathrm{R}}}}{\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}} + \frac{\beta_{\mathrm{EU}}\,\tau}{\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}} = \lambda^* + \frac{\beta_{\mathrm{EU}}\,\tau}{\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}}. \tag{7}$$

□

From Prop. 3, at the lower level of (6), $J^*_{\text{EU}}(\tau) = \frac{\alpha_{\text{EU}}^2}{2\beta_{\text{EU}}} - \alpha_{\text{EU}}\lambda^t(\tau) + \frac{\beta_{\text{EU}}}{2}\left(\lambda^t(\tau)^2 - \tau^2\right)$, $J^*_n(\tau) = \frac{b_n^2}{4a_n} - c_n - \left(\bar{d}_n + \frac{b_n}{2a_n}\right)\lambda^t(\tau) + \frac{\lambda^t(\tau)^2}{4a_n}, \forall n \in \{A, R\}$. The tariff value τ is determined by optimizing the upper-level utility function $J_{\text{EU}}(\boldsymbol{x}_{EU}, \tau)$. It is obtained by substituting $\lambda^t(\tau)$ from Equation (7) in the EU's utility function, which gives a quadratic function in τ. At a Stackelberg Equilibrium (SE), $\frac{\partial J_{\text{EU}}(\boldsymbol{x}_{\text{EU}}, \tau)}{\partial \tau}|_{\tau=\tau^*} = 0$, we obtain $\tau^* = \frac{\beta_{\text{EU}}\left(\beta_{\text{EU}} + \frac{1}{2\,a_{\text{A}}} + \frac{1}{2\,a_{\text{R}}}\right)}{(\beta_{\text{EU}} + \frac{1}{2a_{\text{A}}} + \frac{1}{2a_{\text{R}}})^2 - \beta_{\text{EU}}^2}\lambda^* - \frac{\alpha_{\text{EU}}\left(\beta_{\text{EU}} + \frac{1}{2\,a_{\text{A}}} + \frac{1}{2\,a_{\text{R}}}\right)}{(\beta_{\text{EU}} + \frac{1}{2a_{\text{A}}} + \frac{1}{2a_{\text{R}}})^2 - \beta_{\text{EU}}^2}$. By replacing τ^* by its value in Equation (7), we obtain:

$$\lambda^t = \lambda^t(\tau^*) = \frac{\left(a_{\text{A}} + a_{\text{R}} + 2\,a_{\text{A}}\,a_{\text{R}}\,\beta_{\text{EU}}\right)^2}{\left(a_{\text{A}} + a_{\text{R}}\right)\,\left(a_{\text{A}} + a_{\text{R}} + 4\,a_{\text{A}}\,a_{\text{R}}\,\beta_{\text{EU}}\right)}\,\lambda^* - \frac{4\,a_{\text{A}}^2 a_{\text{R}}^2\,\alpha_{\text{EU}}\,\beta_{\text{EU}}}{\left(a_{\text{A}} + a_{\text{R}}\right)\left(a_{\text{A}} + a_{\text{R}} + 4\,a_{\text{A}}\,a_{\text{R}}\,\beta_{\text{EU}}\right)}.$$

In order to analyze such a tax policy's impact on EU, Asia and Russia's utility, we can establish the following relationships:

Theorem 1 *The markets' utility at a SE of $\mathcal{G}_\tau$ can be expressed as affine functions in their values in $\mathcal{G}$: $J^t_n = A^t_n J^*_n + B^t_n, \forall n \in \{EU, A\}$, $J^t_R = -3A^t_R J^*_R + B^t_R$, where A^t_n is constant, B^t_n depends linearly on λ^*, and J^*_n is quadratic in λ^*, $\forall n$.*

Proof. The result is an application of the polynomial Euclidean division; details are given in [7].

□

In the price-maker model defined in Section 3.3, we obtain results similar to Proposition 6 and Theorem 1. The tariff τ_{v} being fixed, let $\mathcal{G}^{\text{v}}_{\tau_{\text{v}}}$ be the resulting game and SOL($\mathcal{G}^{\text{v}}_{\tau_{\text{v}}}$) denote the set of solutions of $\mathcal{G}^{\text{v}}_{\tau_{\text{v}}}$. The Stackelberg game takes the form:

$$\begin{aligned}
&\max_{\tau_{\text{v}}, \boldsymbol{x}} \quad J^{\text{v}}_{\text{EU}}\,(x_{\text{EU}}, \tau_{\text{v}}),\\
&\text{s.t} \quad \forall n \in \mathcal{N}, \ \max_{\boldsymbol{x}_n} \quad J^{\text{v}}_{\text{n}}(\boldsymbol{x}_{\text{n}}),\\
&\qquad\qquad\qquad \text{s.t.} \quad \boldsymbol{x}_{\text{n}} \in \mathcal{X}_{\text{n}}(\boldsymbol{x}_{\text{-n}}).
\end{aligned} \tag{8}$$

Proposition 7 *The market price in $\mathcal{G}^v_{\tau_v}$ can be expressed as an affine function of the market price in $\mathcal{G}^v$, $\lambda^t_v(\tau_v) = \lambda^* - \frac{1}{\beta_{EU} + \frac{1}{2\,a_A} + \frac{1}{2\,a_R}} \cdot \frac{\tau_v}{2a_R}$. Furthermore, there exists a unique tariff τ^*_v solution of the Stackelberg game* (8).

Proof. This is easily demonstrated by proceeding in a similar way to the proof of Proposition 6; details are given in [7]. □

At the lower level of (8), all the VEs give the same values for the utility functions at a VE: $J^{\text{v}^t}_{\text{EU}}(\tau_{\text{v}}) = -\frac{\beta_{\text{EU}}}{2}\lambda^{t^2}_{\text{v}}(\tau_{\text{v}}) + \frac{\alpha_{\text{EU}}^2}{2\,\beta_{\text{EU}}}$, $J^{\text{v}^t}_{\text{A}}(\tau_{\text{v}}) = -\frac{1}{4\,a_{\text{A}}}\lambda^{t^2}_{\text{v}}(\tau_{\text{v}}) + \frac{b_{\text{A}}^2}{4\,a_{\text{A}}} - c_{\text{A}}$, $J^{\text{v}^t}_{\text{R}}(\tau_{\text{v}}) = -\frac{1}{4\,a_{\text{R}}}\lambda^{t^2}_{\text{v}} + \left(\frac{2\,b_R - 2\,\tau_{\text{v}}}{4\,a_{\text{R}}} - \frac{b_{\text{R}}}{2\,a_{\text{R}}}\right)\lambda^t_{\text{v}} + \frac{b_{\text{R}}\,(b_R - \tau_{\text{v}})}{2\,a_{\text{R}}} - \frac{(b_{\text{R}} - \tau_{\text{v}})^2}{4\,a_{\text{R}}} - c_{\text{R}}$. The tariff value τ_{v} is determined by optimizing the upper-level utility function $J^{\text{v}}_{\text{EU}}(\boldsymbol{x}_{\text{EU}}, \tau_{\text{v}})$.
It is obtained by substituting $\lambda^t_{\text{v}}(\tau_{\text{v}})$ in the EU's utility function, which gives a quadratic function in τ_{v}. At a Stackelberg Equilibrium (SE), $\frac{\partial J_{\text{EU}}(\boldsymbol{x}_{\text{EU}}, \tau_{\text{v}})}{\partial \tau_{\text{v}}}|_{\tau_{\text{v}}=\tau^*_{\text{v}}} = 0$, we obtain $\tau^*_{\text{v}} = 2\,a_{\text{R}}(\alpha_{\text{EU}} + \bar{d}_{\text{A}} + \bar{d}_{\text{R}} + \frac{b_{\text{A}}}{2\,a_{\text{A}}} + \frac{b_{\text{R}}}{2\,a_{\text{R}}})$. By replacing τ^*_{v} by its value in $\lambda^t_{\text{v}}(\tau_{\text{v}})$, we obtain: $\lambda^t_{\text{v}} = \lambda^t_{\text{v}}(\tau^*_{\text{v}}) = 0$.

Theorem 2 *The markets' utility at a SE solution of $\mathcal{G}^v_{\tau_v}$ can be expressed as affine functions in the markets' utility at a NE solution of $\mathcal{G}^v$: $J^{v^t}_n = A^{v^t}_n J^{v^t}_n + B^{v^t}_n, \forall n \in \{EU, A\}$, $J^t_R = A^{v^t}_R J^*_R + B^{v^t}_R$, where $A^{v^t}_n$ is a constant, $B^{v^t}_n$ depends linearly on λ^*, and J^*_n is quadratic in λ^*, $\forall n$.*

Proof. The result is an application of the polynomial Euclidean division [7]. □

3.4 Optimum Sanction

We start with *the price-taker model*. The aim of the sanction is to maximize the EU's objective function and minimize Russia's through a ρ parameter. The optimum sanction model is similar to the optimum tariff model (6), except that the upper level objective function is replaced by $\widetilde{J}(\boldsymbol{x}_{\mathrm{EU}}, \boldsymbol{x}_{\mathrm{R}}, \tau) \stackrel{\text{def}}{=} J_{\mathrm{EU}}(\boldsymbol{x}_{\mathrm{EU}}, \tau) + \rho J_{\mathrm{R}}(\boldsymbol{x}_{\mathrm{R}}, \tau)$. The optimal value of τ is obtained from $\frac{\partial \widetilde{J}(\boldsymbol{x}_{\mathrm{EU}}, \boldsymbol{x}_{\mathrm{R}}, \tau)}{\partial \tau}|_{\tau=\tau^s} = 0$, which gives $\tau^s = \frac{\frac{\beta_{\mathrm{EU}}^2}{D_2} - \frac{3\,\beta_{\mathrm{EU}}\rho}{2\,a_{\mathrm{R}}\,D_2}}{D_1}\lambda^* - \frac{\frac{\alpha_{\mathrm{EU}}\,\beta_{\mathrm{EU}}}{D_2} + \frac{\rho\beta_{\mathrm{EU}}\left(\bar{d}_{\mathrm{R}} - \frac{b_{\mathrm{R}}}{2\,a_{\mathrm{R}}}\right)}{D_2}}{D_1}$, where $D_1 \stackrel{\text{def}}{=} \beta_{\mathrm{EU}} - \frac{\beta_{\mathrm{EU}}^3}{D_2^2} + \frac{3\,\beta_{\mathrm{EU}}^2\,\rho}{2\,a_{\mathrm{R}}\,D_2^2}$ and $D_2 \stackrel{\text{def}}{=} \beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}$.

Theorem 3 *There exists an affine relationship between τ^s in the optimum sanction model and τ^* in the optimum tariff model: $\tau^s = \widetilde{A}_s\tau^* + \widetilde{B}_s$, where $\widetilde{A}_s$ and $\widetilde{B}_s$ are constants defined in [7]. A special case is when ρ=0, leading to $\tau^s = \tau^*$.*

Proof. Assuming $y_1 = a_1x + b_1$ and $y_2 = a_2x + b_2$, when we do $\frac{y_2}{y_1}$, we can write $y_2 = C_1y_1 + C_2$. Thus, $C_1 = \frac{a_2}{a_1}$ and $C_2 = b_2 - \frac{a_2 * b_1}{a_1}$. By proceeding in this way, we obtain coefficients $\widetilde{A}_s$ and $\widetilde{B}_s$. When $\rho = 0$, we can easily see that $\widetilde{A}_s = 1$ and $\widetilde{B}_s = 0$ then $\tau^s = \tau^*$. □

We obtain $\lambda^s(\tau) = \lambda^t(\tau)$. At $\tau = \tau^s$ we obtain the following expression for the market price in $\mathcal{G}_{\tau^s}$: $\lambda^s(\tau^s) = \left(\frac{\beta_{\mathrm{EU}}\left(\frac{\beta_{\mathrm{EU}}^2}{E_2} - \frac{3\,\rho\,\beta_{\mathrm{EU}}}{2\,a_{\mathrm{R}}\,E_2}\right)}{E_1} + 1\right)\lambda^* - \frac{\beta_{\mathrm{EU}}\left(\frac{\alpha_{\mathrm{EU}}\,\beta_{\mathrm{EU}}}{E_2} + \frac{\rho\beta_{\mathrm{EU}}\left(\bar{d}_{\mathrm{R}} - \frac{b_{\mathrm{R}}}{2\,a_{\mathrm{R}}}\right)}{E_2}\right)}{E_1}$,
where $E_1 \stackrel{\text{def}}{=} E_2\left(\beta_{\mathrm{EU}} - \frac{\beta_{\mathrm{EU}}^3}{E_2^2} + \frac{3\,\rho\,\beta_{\mathrm{EU}}^2}{2\,a_{\mathrm{R}}\,E_2^2}\right)$ and $E_2 \stackrel{\text{def}}{=} \beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}$.
Similarly to optimum tariff model in Section 3.4, the geographic markets' utility functions take the same value in each SE [7]. The optimum sanction model can be seen as a generalization of the optimum tariff model; indeed when $\rho = 0$, the regulator's utility function under optimal sanction coincides with that under optimal tariff.

Theorem 4 *There exists an affine relationship between the markets' utility in the optimum sanction model and the markets' utility in the optimum tariff model: $J_n^s = \widetilde{A}_n^s J_n^t + \widetilde{B}_n^s, \forall n \in \mathcal{N}$, where $\widetilde{A}_n^s$ and $\widetilde{B}_n^s$ are some constants defined in [7].*

Proof. Details are given in [7]. □

We continue with the *price-maker model*, and obtain results similar to Theorems 3 and 4. The optimum sanction model is similar to the optimum tariff model (8), except that the upper-level objective function is replaced by $\widetilde{J}_{\mathrm{v}}(\boldsymbol{x}_{\mathrm{EU}}, \boldsymbol{x}_{\mathrm{R}}, \tau_{\mathrm{v}}) \stackrel{\text{def}}{=} J_{\mathrm{EU}}^{\mathrm{v}}(\boldsymbol{x}_{\mathrm{EU}}, \tau_{\mathrm{v}}) + \rho J_{\mathrm{R}}^{\mathrm{v}}(\boldsymbol{x}_{\mathrm{R}}, \tau_{\mathrm{v}})$. The optimal value of τ_{v} is obtained from $\frac{\partial \widetilde{J}_{\mathrm{v}}(\boldsymbol{x}_{\mathrm{EU}}, \boldsymbol{x}_{\mathrm{R}}, \tau_{\mathrm{v}})}{\partial \tau_{\mathrm{v}}}|_{\tau_{\mathrm{v}}=\tau_{\mathrm{v}}^s} = 0$, which gives

$$\tau_{\mathrm{v}}^{\mathrm{s}} = \frac{\rho\left(\frac{1}{4\,a_{\mathrm{R}}^2\left(\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}\right)} - \frac{1}{2\,a_{\mathrm{R}}}\right)}{\widetilde{D}_1}\lambda^* + \frac{\beta_{\mathrm{EU}}\left(\alpha_{\mathrm{EU}} + \bar{d}_{\mathrm{A}} + \bar{d}_{\mathrm{R}} + \frac{b_{\mathrm{A}}}{2\,a_{\mathrm{A}}} + \frac{b_{\mathrm{R}}}{2\,a_{\mathrm{R}}}\right)}{2a_{\mathrm{R}}\,\widetilde{D}_1\left(\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}\right)^2},$$

where $\widetilde{D}_1 \stackrel{\text{def}}{=} 2\rho\left(\frac{1}{16\,a_{\mathrm{R}}^3\left(\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}\right)^2} - \frac{1}{4\,a_{\mathrm{R}}^2\left(\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}\right)} + \frac{1}{4\,a_{\mathrm{R}}}\right) + \frac{\beta_{\mathrm{EU}}}{4\,a_{\mathrm{R}}^2\left(\beta_{\mathrm{EU}} + \frac{1}{2\,a_{\mathrm{A}}} + \frac{1}{2\,a_{\mathrm{R}}}\right)^2}$.

Remark 1. Theorems 3, 4 can easily be extended to the variational setting of the price-maker model [7]. Furthermore, we infer that there exists a closed-form mapping linking the markets' utility at a SE under optimal tariff/sanction to their utility at a NE solution of $\mathcal{G}$.

Correlated equilibrium could be an alternative way to interpret optimal taxation as modifications imposed by a regulator in the agents' strategies.

4 Dealing with Bounded Rationality

In a realistic setting, gas demand is influenced by international political tensions, economic growth, geographical location, and weather conditions, all of which are unpredictable factors. This creates uncertainty about the availability of this commodity, which can vary over time. To hedge against the risk of supply shortage, markets may behave irrationally when defining trades and market prices. To model the irrational behavior of economic agents, we use Cumulative Prospect Theory (CPT). CPT is an extension of Prospect Theory (PT), developed to remedy the latter's inability to capture nonlinear attitudes to risk and probability weighting effects.

4.1 Prospect Theory-Based Game Formulation

More formally, let $(\Xi, \mathcal{A}, \mathbb{P})$ denote a complete measure space and let $\xi \in \Xi$ be the stochastic EU's gas demand. For a subset $\mathcal{A}$ of the real, we define the indicator function as $\mathbb{1}_{\mathcal{A}}(y) = 1$ if $y \in \mathcal{A}$; 0 otherwise. In line with [4], we model the value function $\upsilon : \mathbb{R} \to \mathbb{R}$ describing the (behavioral) value of gains or losses (i.e., the loss aversion) as a piecewise quadratic function:

$$\upsilon(y) = (y - \varsigma y^2)\mathbb{1}_{\{y \geq 0\}} + \kappa(y + \varsigma y^2)\mathbb{1}_{\{y<0\}}, \tag{9}$$

where the parameters $1 \leq \kappa, \varsigma \leq 1$ define the loss aversion tendency in gain and risk-seeking tendency in loss of a decision-maker. Here, $y \stackrel{\text{def}}{=} y(\xi)$ denotes a measurable function of the uncertain demand ξ, hence it is itself a random variable.

CPT captures the way agents assign a subjective value to the occurrence probabilities of an event when making decisions under uncertainty. Specifically, the expectation of (9) is no longer taken with respect to the *true* event probabilities, $p = \mathbb{P}(y \geq 0)$ and $1 - p = \mathbb{P}(y < 0)$, but with respect to a *distorted* probability that typically exacerbates the probability of extreme events. Following [9], we model the probability distortion function as:

$$w(p) = w_+(p)\mathbb{1}_{\{x \geq 0\}} + w_-(p)\mathbb{1}_{\{x<0\}}, \tag{10}$$

where $w_+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$, $w_-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}$, for $0 < \gamma$, $\delta \leq 1$, and w continuous.

We extend the Expected Utility Theory (EUT) framework from Section 3 assuming bounded rationality of EU towards gas supply, by introducing the probability distortion $\mathbb{Q} \stackrel{\text{def}}{=} w_\sharp \mathbb{P}$ where w is defined in 10. We let ξ be the random variable measuring EU's gas demand and we set $S(\xi) = \frac{1}{\beta_{\text{EU}}}\left(\alpha_{\text{EU}}\xi - \frac{\xi^2}{2}\right)$. For simplicity, under EUT, we assume $\xi \sim \mathbb{P}$ where $\mathbb{P}$ is defined on a finite number of atoms $\Xi \stackrel{\text{def}}{=} \{\xi_k\}_{k=1}^K$ with $p_k \stackrel{\text{def}}{=} \mathbb{P}[\xi = \xi_k]$, $\forall k = 1, ..., K$. Moreover, we define $R(\xi) \stackrel{\text{def}}{=} \xi - Q_{\text{EU}}$ as the stochastic supply shortage. The value function of interest is as in (9) with $y(\xi) = S(\xi) - R(\xi)$. Note that $S(\xi) < R(\xi)$ can be interpreted as a supply shortage. The EU's utility function becomes stochastic giving $J_{\text{EU}}(\boldsymbol{x}_{\text{EU}}, \xi) = \upsilon(\xi) - p_{\text{EU,R}}q_{\text{R,EU}} - p_{\text{EU,A}}q_{\text{A,EU}}$ for the price-taker structure and $J^{v}_{\text{EU}}(\boldsymbol{x}_{\text{EU}}, \xi) = \upsilon(\xi)$ in the price-maker structure. EUT and PT-based games are formulated taken the expectation of J_{EU} and J^{v}_{EU} with respect to ξ. Under EUT, $\mathbb{E}_{\mathbb{Q}}[\upsilon(\xi)] = \sum_k \upsilon(\xi_k)p_k$; while under PT we get:

$$\mathbb{E}_{\mathbb{Q}}[\upsilon(\xi)] = \int_{\text{supp}(\mathbb{Q})} \upsilon(\xi)d\mathbb{Q}(\xi) = \sum_k \Big[\Big(S(\xi_k) - R(\xi_k) - \varsigma\,(S(\xi_k) - R(\xi_k))^2\Big)\, w_+(p_k)$$
$$+\kappa\Big(S(\xi_k) - R(\xi_k) + \varsigma\,(S(\xi_k) - R(\xi_k))^2\Big)\, w_-(p_k)\Big].$$

The objective functions of Asia and Russia remain the same as in Section 3.

Remark 2. Theorems 1-4, Propositions 1-7 can be extended to the PT game almost everywhere (a.e.) given that the conservative gradient equals the standard gradient almost everywhere [3].

4.2 Price of Irrationality

Drawing a parallelism with the well-known concept of Price of Anarchy, we introduce the concept of Price of Irrationality (PoI). Formally, the PoI measures the loss of efficiency due to the bounded rationality of the agents. Let $\boldsymbol{x}^*$ and y^* be solutions of the EUT and PT-game resp., we define the PoI as: $\text{PoI}(\mathcal{G}) \stackrel{\text{def}}{=} \frac{\sum_i \mathbb{E}_{\mathbb{P}}[J_i(\boldsymbol{x}_i^*,\xi)]}{\sum_i \mathbb{E}_{\mathbb{P}}[J_i(\boldsymbol{y}_i^*,\xi)]}$. Since the PT-based equilibrium is based on subjective prospects, it includes smaller profits than EU-based equilibrium, thus $\text{PoI}(\mathcal{G}) \geq 1$.

Theorem 5 *$PoI(\mathcal{G}) = \frac{1}{\sum_i \mathbb{E}_{\mathbb{P}}[J_i(\boldsymbol{x}_i,\xi)]}\left[A_c {\lambda^*_{PT}}^2 + B_c \lambda^*_{PT} + C_c\right]$ a.e., where A_c, B_c, C_c are resp. the sum of the coefficients of the markets' utility monomials under bounded rationality and λ^*_{PT} is the export price in the PT-based game.*

Proof. Market i's expected utility evaluated at a NE of $\mathcal{G}$ can be written as a quadratic form which depends on λ^* under EUT and λ^*_{PT} under PT. Thus, the PoI can be expressed as a quadratic form in λ^*_{PT}. Closed-form expressions are detailed in [7]. □

Similar reasonings apply to $\text{PoI}(\mathcal{G}_\tau)$, $\text{PoI}(\mathcal{G}_{\tau^s})$ and price-taker structures.

4.3 Numerical Results

We implement the PT-based game on a randomized instance; details about the model parameters are given in [7].In Figure 1, we have represented the utility of EU, Asia, and Russia assuming bounded rationality of the agents for price-taker (a), (b) and price-maker (c), (d) models. Under both formulations, we observe that increasing the sanction towards Russia, increases the EU's welfare and severely decreases Russia's utility. However, Asia is also severely penalized by this sanction, which can be explained by the rise of the export price.

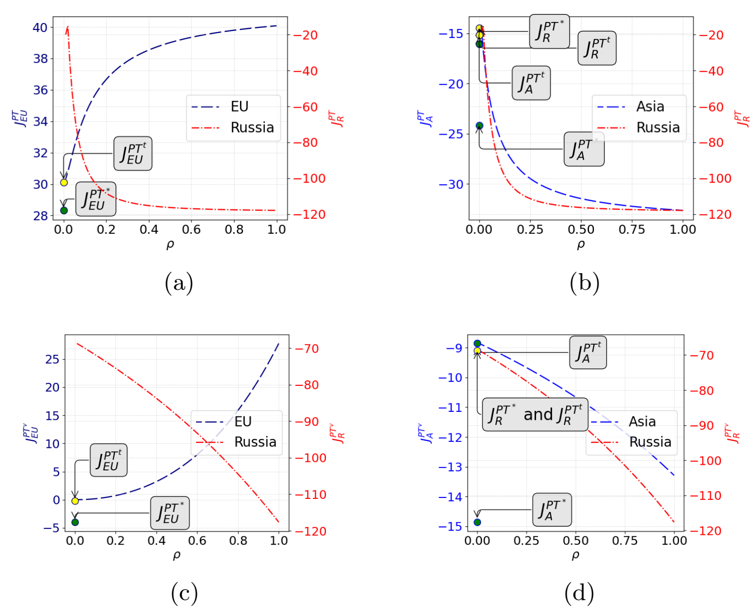

Fig. 1: PT-game markets' utility for (a), (b) price-taker agents, (c), (d) price-maker agents.

In Figure 2, we have plotted the PoI for price-taker (a) and price-maker (b) formulations, for a fixed set of PT parameters. We observe that: the less risky the situation is, the more close

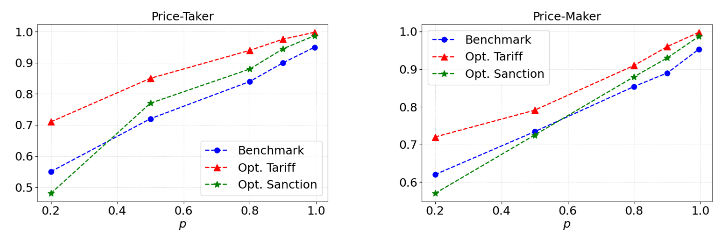

Fig. 2: PoI for (a) price-taker formulation, (b) price-maker formulation as a function of p.

to rationality the stakeholders should behave, and vice-versa; optimum tariff always achieves higher PoI, thus guaranteeing outcomes closer to the rational setting; there exists a threshold $p = 0.5$ in (a) and $p = 0.53$ in (b) above which optimum sanction achieves higher PoI than the benchmark though remaining below the optimum tariff. This can be explained by the higher difficulty to coordinate stakeholders under the threat of a sanction.

5 Conclusion

We consider a finite number of geographic markets involved in gas trading modeled as a non-cooperative game assuming either price-taker or price-maker structure. We consider three settings: (i) a benchmark model involving neither tariff nor sanction; (ii) optimal tariff; (iii) optimal sanction, both on the imports. Assuming demand is uncertain, we capture the bounded rationality of stakeholders with respect to a potential risk of supply shortage using Cumulative Prospect Theory. To analyze the outcome of this PT-based game, we introduce a new metric, the Price of Irrationality (PoI), to measure the resulting efficiency loss. On a random instance, we observe that the less risky the situation is, the closer to rational the stakeholders are, and vice-versa. Furthermore, the optimum tariff achieves higher PoI, thus guaranteeing outcomes closer to the rational setting. In an extension, black markets could be considered.

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