On the stability of DAG-based distributed ledger with heterogeneous delays

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Abstract. Directed Acyclic Graphs (DAGs) are an appealing design for Distributed Ledger (DL) architectures. Specifically, DAG-based DLs offer valuable benefits compared to blockchains, such as improved scalability, lightweight consensus mechanisms and lower transaction costs. However, due to the communication delays and distributed nature of the DL, some transactions may remain unapproved. Previous works have provided bounds on the expected number of unapproved transactions when the transactions selection strategy is uniform. Still, a transaction should be preferably validated by multiple nodes so as to increase the trust in the ledger. In this paper, we introduce a bound on the expected number of transactions that are approved by at most one node. For this purpose, we define a new stochastic model based on stochastic sets, which captures the evolution of DAG-based DL. The proposed model enables us to establish a quadratic bound on the drift in the number of tips. We then demonstrate that the expected volume of transactions validated by at most one node is bounded. These results indicate that the occurrence of large volume of transactions validated by at most one node happens with sufficiently low probability.

Keywords: Distributed ledger · DAG-based distributed ledger · Stochastic Process.

1 Introduction

Distributed ledger technology (DLT) is a decentralized peer-to-peer digital system that records simultaneously transactions between multiples parties spread across different locations. By relying on advanced cryptography and consensus mechanisms, DLT enables participants to maintain a consistent and immutable ledger. This eliminates the need for a centralized data store and central authority, unlike traditional databases. While DLT (i.e., blockchain) has showed significant potential to support financial transactions (and in particular Bitcoin crypto-currency), broader and more mainstream adoption is likely to follow if scalability issue (i.e., difficulties of handling large volume of transactions quickly and efficiently) is resolved. In this regards, DLTs based on a Directed Acyclic Graph (DAG) data structure have been introduced. DAG consists of a number vertices (i.e., transactions) connected by directed edges such that there is no cycle (i.e., directed path connecting a vertex to itself). DLT evolves over time as follows: a node that creates a new transaction is required to validate (hereafter also referred to as approve or verify) two previous transactions using a transaction selection mechanism introduced in [10]. Few selection strategies have been introduced, among which the random selection algorithm, wherein transactions are randomly validated (for example uniformly, using Markov chain Monte Carlo algorithm [3]). There is a growing trend among companies and startups, such as IOTA [9], to integrate DAG-based DLT into their business operations. A key advantage of DAG-based DLTs over blockchains is their ability to connect to any current vertex in the graph, allowing new transactions (i.e., messages) to be added to the ledger more efficiently and rapidly. Although there are undeniable benefits compared to classical blockchain, DAG-based DLTs have drawbacks. In particular, due to the distributed and asynchronous nature of the DLT, the verification process induces latency and inconsistencies: for instance, a node may not consider the same DL state, may believe that a validated transaction is not yet validated and may be unaware of the existence of a new transaction. As a consequence, the ledger performance could be significantly affected in the sense that an increased number of transactions remains unapproved. This raises the question of whether the number of unapproved transactions, which is called a *tip* in the literature, remains bounded. In this regards, several works [2, 11, 9, 12, 5] have studied the stability of DAG-based DLs, providing bounds on the expected number of tips (i.e. transactions that have not been approved by any node) and the expected validation time. In this paper, we assume that every incoming transaction needs to be approved (at least) two times because nodes are more likely to trust a DL in which transactions are confirmed by multiple independent nodes. Two scenarios are possible: either a node validates a transaction that has never been validated before, or it validates a transaction that has already been validated by another (single) node. For this purpose, we choose to use a multinomial distribution for the tip selection mechanism. This is especially important in financial systems and applications that require a high degree of confidence. In order to estimate the volume of transactions that are insufficiently verified, i.e., not approved by any node or approved by a single node, we provide a bound on the expected number of transactions that have been approved by at most one node, considering the situation in which communication delays are bounded and heterogeneous. In the following, we introduce a new definition of a tip that corresponds to a transaction validated by at most one node. We propose a new stochastic model based on stochastic sets to study the evolution of the number of tips. Then, we determine a negative upper bound on the drift of the number of tips, which allows us to conclude that its expected number is bounded over time.

1.1 Related work

Our review of the literature is focused on studying the stability and performance of DAG-based distributed ledgers (DL). The initial mathematical model

[9] of DAG-based ledgers assumes that a central node manages the ledger, while other nodes access it by requesting information to the central node. The communication delays caused by the interaction between the central server and the other nodes are assumed to be constant. Another assumption is that the number of unapproved transactions remains close to its average value at a constant rate. Based on the same model [9] and on the same conjuncture, some empirical simulation-based studies [4, 6] evaluate the performance of DAG-based ledgers. In [8], The authors assume that multiple distinct message categories exist, which lead to varying processing times and consequently different reception delays. The authors prove a mathematical result under different delays, which arises from various classes of messages. In [1], the authors assume that the strategy for selecting unapproved messages is not uniform and prove that the number of unapproved messages converges to a partial differential equation. In [12], using Markov chain, the authors propose modeling the evolution of DAG-based distributed ledgers with homogeneous delays among nodes. Our paper extends the model proposed by [2], which assumes heterogeneous delays and introduces a mathematical framework to prove that the expected number of unapproved messages is bounded. Most of these works consider a uniform tip selection algorithm and focus on the stability and performance of DAG-based DL, which are evaluated based on the evolution of unapproved messages. Among all these works, none study the stability properties considering the evolution of the number of transactions approved by at most one node in the presence of heterogeneous delays with a non-uniform selection strategy.

2 Mathematical model

For $I \in \mathbb{N}^*$, let $\mathcal{I} = \{1, ..., I\}$ be the set of nodes in the DL. We assume that the state of the DL progresses at discrete time steps $n \in \mathbb{N}$. For $i \in \mathcal{I}$, let C_n^i denote the set of new messages sent by node i at time n. Each node i generates r_i new messages at any time n. We denote $V_n^{i,0}$ as the set of messages that node i views as unapproved at time n, $V_n^{i,1}$ as the set of messages that node i considers approved by a single node $j \in \mathcal{I}(j \neq i)$ at time n, and W_n^i as the set of messages that node i considers either unapproved or approved by a single node (other than i) at time n. Given that the selection algorithm of the IOTA Tangle requires any incoming transaction to approve at least two tips (and can approve the same tip twice), we assume that each new message from node i approves two messages chosen randomly with multinomial distribution, independently and with replacement, in W_n^i. Therefore, a message can potentially validate the same message in W_n^i more than once. Additionally, a message in W_n^i might be approved by multiple messages from C_n^i. Let D_n^i be the set of approved messages by node i at time n and let $\mathcal{D}_n^i$ be the set of all approved messages by node i up to and including time n. That is,

$$\mathcal{D}_n^i = \bigsqcup_{t=0}^{n} D_n^i.$$

We assume that $V_0^{i,0} = \{0\}$, $V_0^{i,1} = D_0^i = C_0^i = \emptyset$ and that a message cannot be validated by the same node at two different times. We define Γ^i_{n,d^i} as the set that contains all messages viewed by i as approved by at least two different nodes at time $n+1$. Node j is informed about the new set of unapproved messages with d^{ij} units of time after n and also about that messages in Γ^i_{n,d^i} should not be approved after d^{ij} units of time from n. Assume that $d^{ij} = d^{ji}$. The evolution of the set Γ^i_{n,d^i} can be modeled as follows :

$$\Gamma^i_{n,d^i} = \bigcup_{j\in\mathcal{I}} \mathcal{D}^j_{n-d^{ij}} \bigcap \left(\bigcup_{l\neq j} \mathcal{D}^l_{n-d^{il}} \right),$$

where $d^{ij} \in \mathbb{N}$ is the delay for node j to observe the new messages sent by node i and the messages approved by node i and $d^i = (d^{ij})_{j\in\mathcal{I}}$. For example, if $s \in \Gamma^i_{n,d^i}$, there exist $k, l \in \mathcal{I}$, such that $s \in \mathcal{D}^k_{n-d^{ik}}$ and $s \in \mathcal{D}^l_{n-d^{il}}$. That is, s was validated by node k between time 1 and time $n - d^{ik}$ and by node l between time 1 and time $n - d^{il}$. Note that, if s was validated at time $n - d^{il}$, it will only be visible to i at time $n+1$ and not at time n. The set W^i_n is given by the following formula:

$$W^i_{n+1} = \bigcup_{j\in\mathcal{I}} \bigcup_{t=0}^{n-d^{ij}} C^j_t - \Gamma^i_{n,d^i}.$$

Note that a node i can only validate a message once. That is the messages validated only by i are no longer tips for i but are still tips in the system. We define Γ^i_n as the set of all approved messages at least by two different nodes including i until time n and Γ_n as the set of all approved messages at least by two different nodes until time n. Then,

$$\Gamma^i_n = \mathcal{D}^i_n \bigcap \left(\bigcup_{j\neq i} \mathcal{D}^j_n \right),$$

and

$$\Gamma_n = \bigcup_{j\in\mathcal{I}} \Gamma^j_n.$$

Let Y_n be the set of messages which are actually tips at time n. Then, Y_n is depicted by

$$Y_n := \left(\bigcup_{t=0}^{n} C_t \right) - \Gamma_n$$

and can be computed recursively as

$$Y_n = C_n \bigsqcup \left(Y_{n-1} - \Gamma_n\right),$$

where $C_n = \bigcup_{i \in \mathcal{I}} C_n^j$. We also need to consider the set A_n of messages that were created by time $(n - d^*)$ and are tips at time n. That is

$$A_n := \bigcup_{t=0}^{n-d^*} C_t - \Gamma_n, \tag{1}$$

where $d^* = \max_{i,j \in \mathcal{I}} d^{ij}$.

Example 1. If $d = 0$ and $\mathcal{I} = \{1, 2, 3\}$, then

- $V_n^{i,0} = V_n^{j,0}$ and $V_n^{i,1} = V_n^{j,1}$ for each $i, j \in \{1, 2, 3\}$.
- The set $\Gamma_n^1 = \Gamma_{n,d^i}^i$ is the set of all messages viewed by i as approved messages by at least two different nodes at time $n + 1$ for each $i, j \in \{1, 2, 3\}$. That is Γ_n^i is the set of validated messages by $\{1, 2\}$, $\{2, 3\}$ or $\{1, 3\}$.
- $W_n^i = Y_n$, for each $i \in \{1, 2, 3\}$.

In this work, we use a multinomial random tip selection strategy. In fact, this strategy is chosen for its ability to handle multiple outcomes with different probabilities. Specifically, transactions in $V_n^{i,0}$ are more likely to be chosen by node i than transactions in $V_n^{i,1}$. We assume that transactions in $V_n^{i,0}$ have a two-thirds chance of being validated by i, while transactions in $V_n^{i,1}$ have a one-third chance. In other words, if $r_i \in \mathbb{N}^*$ is the number of new messages sent by node i at each time n, the probability that a transaction a is validated by a node i at time n is given by

$$\mathbb{P}(a \notin \Gamma_n^i \big| W_n^i) = \begin{cases} 1 & \text{if } a \notin W_n^i \\ \left(1 - \frac{|V_n^{i,0}|}{|W_n^i|}\right)^{r_i^0} \left(1 - \frac{|V_n^{i,1}|}{|W_n^i|}\right)^{r_i^1} & \text{if } a \in W_n^i, \end{cases}$$

where r_i^0 and r_i^1 are the rounded numbers of $\frac{4r_i}{3}$ and $\frac{2r_i}{3}$ respectively. We also define $r := \sum_{i=1}^{\mathcal{I}} r_i$ and $r^0 := \sum_{i=1}^{\mathcal{I}} r_i^0$. In the following section, we are interested in the cardinality X_n of the tips set Y_n at each time step n.

3 Finite bound over the expectation of the cardinality of the tips set

In the following, we introduce a new proof regarding the existence of a finite bound on the expectation of the cardinality of the tips set. Such results demonstrate that DAG-based DLs do not diverge, provided that the delays are bounded. We begin by establishing the following useful properties of the set A_n defined by (1). These properties are needed to prove the bound of the number of tips X_n.

Lemma 1. *Let $i \in \mathcal{I}$ and $n \in \mathbb{N}^*$. Then,*

1. *$A_n \subseteq Y_n$.*
2. *The number of tips $X_n = |Y_n|$ is bounded as follows:*

$$|A_n| \leq X_n \leq |A_n| + rd^*.$$

3. *The cardinal of the set of messages that node i believes to be unapproved or approved by a single node (other than i) at time n, denoted by W_n^i is upper bounded as follows:*

$$|W_{n+1}^i| \leq |A_n| + 3rd^*.$$

4. *We also have that W_n^i is upper bounded by X_n as follows:*

$$|W_{n+1}^i| \leq X_n + 2rd^*.$$

Proof. We will now prove the four statements:

1. Direct from definition of A_n.
2. Note that

$$Y_n = \left(\bigcup_{t=0}^{n} C_t\right) - \varGamma_n \subseteq \left(\bigcup_{t=0}^{n-d^*} C_t - \varGamma_n\right) \bigcup \left(\bigcup_{t=n-d^*+1}^{n} C_t\right) = A_n \bigcup \left(\bigcup_{t=n-d^*+1}^{n} C_t\right),$$

   which implies that $X_n \leq |A_n| + rd^*$.
3. We have

$$\begin{aligned}
W_{n+1}^i &= \bigcup_{j\in\mathcal{I}} \bigcup_{t=0}^{n-d^{ij}} C_t^j - \varGamma_{n,d^i}^i \\
&\subseteq \bigcup_{j\in\mathcal{I}} \bigcup_{t=0}^{n-d^{ij}} C_t^j - \varGamma_{n-d^*} \\
&\subseteq \left(\bigcup_{t=0}^{n-d^*} C_t - \varGamma_{n-d^*}\right) \bigcup \left(\bigcup_{t=n-d^*+1}^{n} C_t\right) \\
&\subseteq \left(\bigcup_{t=0}^{n-d^*} C_t - \varGamma_n\right) \bigcup \left(\bigcup_{t=n-d^*+1}^{n} C_t\right) \bigcup (\varGamma_n - \varGamma_{n-d^*}) \\
&\subseteq \left(\bigcup_{t=0}^{n-d^*} C_t - \varGamma_n\right) \bigcup \left(\bigcup_{t=n-d^*+1}^{n} C_t\right) \bigcup \left(\bigcup_{j\in\mathcal{I}} \left(\bigcup_{t=n-d^*+1}^{n} D_t^j\right)\right),
\end{aligned}$$

$$\begin{aligned}
\Rightarrow |W_{n+1}^i| &\leq |A_n| + d^*r + 2rd^* \\
&= |A_n| + 3rd^*.
\end{aligned}$$

4. Observe that:

$$\begin{aligned}
W^i_{n+1} &= \bigcup_{j\in\mathcal{I}} \bigcup_{t=0}^{n-d^{ij}} C^j_t - \Gamma^i_{n,d^i} \\
&\subseteq \bigcup_{t=0}^{n} C_t - \Gamma_{n-d^*} \\
&\subseteq \left(\bigcup_{t=0}^{n} C_t - \Gamma_n\right) \bigcup (\Gamma_n - \Gamma_{n-d^*}) \\
&\subseteq \left(\bigcup_{t=0}^{n} C_t - \Gamma_n\right) \bigcup \left(\bigcup_{j\in\mathcal{I}} \left(\bigcup_{t=n-d^*+1}^{n} D^j_t\right)\right) \\
&= Y_n \bigcup \left(\bigcup_{t=n-d^*+1}^{n} D_t\right) \\
\Rightarrow |W^i_{n+1}| &\leq X_n + 2rd^*.
\end{aligned}$$

In the following theorem, we prove a bound on the drift of X_n using the above lemma.

Theorem 1. *The drift of the number of tips X_n is bounded as follows*

$$\mathbb{E}(X_{n+1} \big| X_n = x) \leq r + x - \frac{rr^0(x - rd^*)}{x + 2rd^*}\left(1 - \frac{r^0 r}{x + 2rd^*}\right), \tag{2}$$

Moreover, if X_n tends to the infinity, we have

$$\lim_{x\to\infty} \mathbb{E}(X_{n+1} - X_n \big| X_n = x) \leq r - \frac{4}{3}r^2. \tag{3}$$

Furthermore, for $a = 7(rd^* + rr^0)$ and $r^0 \geq 2$

$$\mathbb{E}(X_{n+1} - X_n \big| X_n = x, x \geq a) \leq \frac{-r}{7}. \tag{4}$$

Proof. Since each node acts independently at a given time, the probability that an element s of A_n will be tip at time $n+1$ is given by

$$\begin{aligned}
\mathbb{P}(s \notin \Gamma_{n+1} \big| s \in A_n) &= \prod_{i\in\mathcal{I}} \mathbb{P}(s \notin \Gamma^i_{n+1} \big| s \in A_n) \\
&= \prod_{i\in\mathcal{I}} \left(1 - \frac{|V^{i,0}_n|}{|W^i_n|}\right)^{r^0_i} \left(1 - \frac{|V^{i,1}_n|}{|W^i_n|}\right)^{r^1_i},
\end{aligned}$$

which implies that the expected number of tips in A_n that no longer remain as tips at time $n+1$ is given by

$$\begin{aligned}
\mathbb{E}(|A_n \cap \Gamma_{n+1}| \big| X_n = x) &= |A_n| - |A_n| \prod_{i\in\mathcal{I}} \left(1 - \frac{|V_{n+1}^{i,0}|}{|W_{n+1}^i|}\right)^{r_i^0} \left(1 - \frac{|V_{n+1}^{i,1}|}{|W_{n+1}^i|}\right)^{r_i^1} \\
&\geq |A_n| - |A_n| \prod_{i\in\mathcal{I}} \left(1 - \frac{|V_{n+1}^{i,0}|}{|W_{n+1}^i|}\right)^{r_i^0} \\
&\geq |A_n| - |A_n| \prod_{i\in\mathcal{I}} \left(1 - \frac{r}{x+2rd^*}\right)^{r_i^0} \qquad (5) \\
&= |A_n| - |A_n| \left(1 - \frac{r}{x+2rd^*}\right)^{r^0} \quad (\text{since } r^0 := \sum_{i\in\mathcal{I}} r_i^0) \\
&\geq |A_n| - |A_n| \left(1 - \frac{rr^0}{x+2rd^*} + \frac{r^0(r^0-1)r^2}{2(x+2rd^*)^2}\right) \\
&= |A_n| \left(\frac{rr^0}{x+2rd^*} - \frac{r^0(r^0-1)r^2}{2(x+2rd^*)^2}\right) \\
&= \frac{|A_n|rr^0}{x+2rd^*} \left(1 - \frac{(r^0-1)r}{2(x+2rd^*)}\right) \\
&\geq \frac{|A_n|rr^0}{x+2rd^*} \left(1 - \frac{r^0 r}{x+2rd^*}\right) \\
&\geq \frac{rr^0(x-rd^*)}{x+2rd^*} \left(1 - \frac{r^0 r}{x+2rd^*}\right), \quad (\text{from Lemma 1})
\end{aligned}$$

where (5) follows from Lemma 1 and the fact that $|V_n^{i,0}| \geq r$. Recall that

$$Y_{n+1} = C_{n+1} \bigsqcup (Y_n - \Gamma_{n+1})$$

then

$$X_{n+1} = r + X_n - |Y_n \cap \Gamma_{n+1}|.$$

Taking the conditional expectation on both sides, we obtain

$$\begin{aligned}
\mathbb{E}(X_{n+1} \big| X_n = x) &= r + x - \mathbb{E}(|A_n \cap \Gamma_{n+1}| \big| X_n = x) \\
&\leq r + x - \frac{rr^0(x-rd^*)}{x+2rd^*} \left(1 - \frac{r^0 r}{x+2rd^*}\right)
\end{aligned}$$

which provide (2). The upper bound is coming from the fact that $A_n \subseteq Y_n$ For the proof of (3), it suffices to take the limit as $x \to \infty$ in (2), we obtain

$$\begin{aligned}
\lim_{x\to\infty} \mathbb{E}(X_{n+1} - X_n \big| X_n = x) &\leq r - rr^0 \\
&= r - \frac{4}{3} r^2.
\end{aligned}$$

Let $a = 7(rd^* + rr^0)$. Then,

$$
\begin{aligned}
\mathbb{E}(X_{n+1} - X_n \big| X_n = x, x \geq a) &\leq r - \frac{rr^0(x - rd^*)}{x + 2rd^*}\left(1 - \frac{rr^0}{x + 2rd^*}\right) \\
&\leq r - \frac{2r(x - rd^*)}{x + 2rd^*}\left(1 - \frac{rr^0}{x + 2rd^*}\right) \\
&\leq r - \frac{2r(a - rd^*)}{a + 2rd^*}\left(1 - \frac{rr^0}{a + 2rd^*}\right) \qquad (6) \\
&= r - \frac{2r(6rd^* + 7rr^0)}{9rd^* + 7rr^0}\left(1 - \frac{rr^0}{9rd^* + 7rr^0}\right) \\
&\leq r - \frac{2r(6rd^* + 7rr^0)}{9rd^* + 7rr^0}\left(1 - \frac{rr^0}{7rr^0}\right) \\
&\leq r - \frac{2 * 6r^2d^*}{9rd^*}\frac{6}{7} \\
&= r - \frac{8r}{7} = \frac{-r}{7},
\end{aligned}
$$

where (6)follows from the monotonicity of the functions $\frac{2r(x-rd^*)}{x+2rd^*}$ and $\left(1 - \frac{rr^0}{x+2rd^*}\right)$ in x.

We recall, from Theorem 1 in [7], that if a stochastic process has a negative upper bound on the drift and bounded jumps, then it has a bounded expectation. Hence, from the above results, we can deduce the existence of an upper bound on the expected number of tips X_n.

Corollary 1. *By construction, the process X_n have a bounded jumps. That is,*

$$\mathbb{E}(|X_{n+1} - X_n|^p \big| X_n, ..., X_0) \leq (2r)^p, \qquad \forall p \geq 0.$$

Then, using the inequality (4) and Theorem 1 in [7], we deduce that

$$\mathbb{E}(X_n) < \infty.$$

4 Conclusion

We herein introduce a new definition of tip in DAG-based DL and propose an innovative mathematical model, based on stochastic sets of messages, to describe the behavior of the DAG-based DL considering the existence of heterogeneous delays between nodes. Moreover, we establishe an upper bound on the drift and then derive an upper bound on the expected number of tips.

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